Differentiation – From Chords to Tangents
The Software Tutorial

Learning Outcomes

After using the applet and accompanying tutorial and theory sheets you should

- be aware of the difference between a chord and a tangent
- be able to handle the different notation used for the gradient of a chord and of a tangent
- have developed, through experimentation, an understanding of basic differentiation and how chords and tangents are related to the differentiation process
- be able to answer "what if ...?" questions about basic differentiation.

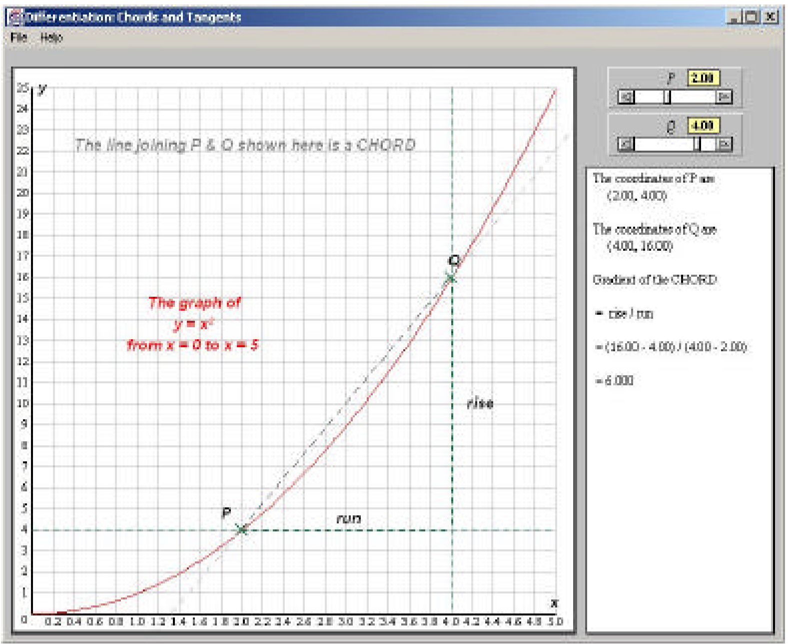
Introduction

Differentiation has important applications in science and technology. Examples include: Planetary motion, vibrations of mass/spring/damper systems and determining maximum and minimum values. Since differentiation is frequently used in modelling time-dependent systems, the independent variable often used is t (for time) rather than x. For example, in a mass/spring/damper system, the system output, the deflection, y, of the mass from its equilibrium position depends on the time, t, that the system has been in motion. This tutorial sheet and accompanying applet use y and x, as often used when meeting differentiation for the first time.

The accompanying applet is used solely to introduce basic concepts of differentiation and, consequently, uses just the one function ($y = x^2$) to put across these ideas.

Loading the Software

Select "Differentiation 1" from the web page from which this tutorial sheet was obtained. When loaded, you will see the "Differentiation1: Chords and Tangents" applet window as shown here.



Running the applet

The two scrollbars on the right of the applet window allow the user to change the x-values of P and Q. Clicking the arrow boxes at either end of the scrollbars effect *small changes* in their values. Clicking in the main part of the box of the scrollbars makes *larger changes*. *Clicking, holding and moving the <u>slider</u> in the slider bar can make **any** sort of change*.

Changing the values by moving the sliders is the only means of user-input.

The Tutorial

The applet shows two points, P and Q, on the graph $y = x^2$ and the (dotted) line that passes through them. When the applet loads, point P is set at 2.0 indicating that its x-coordinate is $x = 2.0$ so the coordinates of P are (2, 4). The default value of Q is 4.0, so point Q has coordinates (4, 16).

[Remember that on $y = x^2$, the y value is the x value squared]

Chords A line that passes *through* two points on a curve is a **chord**, so the dotted line through P and Q is ***the*** chord through P and Q (there is only one chord joining any two points).

Gradient of a straight line The gradient of a straight line is determined by dividing the vertical height that the line rises between any two points on that line by the corresponding horizontal distance between those two points. This is often stated as "rise over run", where the rise and the run can be seen marked on the applet's graphics area. The "rise" is the difference between the y-values of the two points P and Q and the "run" is the difference between the x-values. If the points are labelled P(x_1, y_1) and Q(x_2, y_2) then,

$$\text{gradient of a line} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note the order of subtraction. Here, rise = $y_2 - y_1$ and run = $x_2 - x_1$, but you could have used rise = $y_1 - y_2$ and run = $x_1 - x_2$ to obtain the same gradient. It doesn't matter whether you use $y_2 - y_1$ or $y_1 - y_2$ in the top line as long as the difference in the bottom line is in the same order (both sets of subscripts should be 2 – 1, or both 1 – 2).

Use the information provided by the applet when it initially loads to complete the following table using the coordinates of P as (x_1, y_1) and Q as (x_2, y_2):

P (x_1, y_1)	Q (x_2, y_2)	$y_2 - y_1$	$x_2 - x_1$	$\frac{y_2 - y_1}{x_2 - x_1}$	$y_1 - y_2$	$x_1 - x_2$	$\frac{y_1 - y_2}{x_1 - x_2}$
(2, 4)	(4, 16)						

So the ***gradient of the chord*** joining P(2, 4) and Q(4, 16) is …………………….

Now use the applet, leaving P fixed, to complete the following table:

P	Q	$y_2 - y_1$	$x_2 - x_1$	$\frac{y_2 - y_1}{x_2 - x_1}$
(2, 4)	(4.0, 16)			
(2, 4)	(3.5,)			
(2, 4)	(3.0,)			
(2, 4)	(2.5,)			
(2, 4)	(2.4,)			
(2, 4)	(2.3,)			
(2, 4)	(2.2,)			
(2, 4)	(2.1,)			
(2, 4)	(2.0,)			

[Remember if you want to change the position of Q from $x = 4.0$ to $x = 3.5$ you can either (a) click inside the slider area five times (giving 4.0, 3.9, 3.8, 3.7, 3.6 and then 3.5), (b) click in the scrollbar's left-arrow 50(!) times or (c) move the slider until you obtain 3.5. Of these, (a) is the easiest to use here.]

The entry for the last row might cause you a problem! The $y_2 - y_1$ and $x_2 - x_1$ values are zero; leading to zero divided by zero, which is indeterminate - that is it could be ***any*** value!

Have a look at the sequence of values in the final column of the above table. This column contains the gradients of chords joining P and Q. However, for the last entry P and Q are coincident – they are the same point and ***the chord*** (joining two separate points on the curve) ***becomes the tangent*** (touching at one point on the curve). The missing entry in the final column is the value of ***the gradient of the tangent*** to the curve at the point (2, 4).

So what value should it be? It certainly looks as though it should be the value …….

What about if you continue the table, taking Q to the left of P. Complete the following;

P	Q	$y_2 - y_1$	$x_2 - x_1$	$\frac{y_2 - y_1}{x_2 - x_1}$
(2, 4)	(1.9, 16)			
(2, 4)	(1.8,)			
(2, 4)	(1.7,)			
(2, 4)	(1.6,)			

This, together with the previous table, certainly seems to indicate that the gradient of the tangent of $y = x^2$ at the point P(2,4) is exactly 4.

Unfortunately the applet (deliberately) doesn't allow you any finer adjustment than moving P or Q in steps of 0.1 at a time, so Q at $x = 2.1$ or at $x = 1.9$ is the closest you can get to $x = 2$. However, if you want to try to convince yourself that the gradient of the tangent is 4, use your calculator to complete the following table:

P	Q	$y_2 - y_1$	$x_2 - x_1$	$\frac{y_2 - y_1}{x_2 - x_1}$
(2, 4)	(2.01,)			
(2, 4)	(2.001,)			
(2, 4)	(2.0001,)			
(2, 4)	(2.00001,)			

Convinced?

OK, so the only information obtained from the previous few pages was that the gradient of the tangent of $y = x^2$ at the point P(2,4) is exactly 4. What about the gradient of the tangent at other points? At this stage, I'm afraid all you can do is repeat the process.

Move the point P slider to 3.0 and the Q slider to 5.0 and then complete the following table in the same way as before:

P	Q	$y_2 - y_1$	$x_2 - x_1$	$\frac{y_2 - y_1}{x_2 - x_1}$
(3,)	(5,)			
(3,)	(4.5,)			
(3,)	(4.0,)			
(3,)	(3.5,)			
(3,)	(3.4,)			
(3,)	(3.3,)			
(3,)	(3.2,)			
(3,)	(3.1,)			
(3,)	(3.0,)			

As a good guess, on the basis of what you did in the previous tables, the gradient of the tangent of $y = x^2$ at the point P(3, 9) is ……

Now, place P at $x = 2.5$, move Q either side of P and, without writing entries in a table, note the gradients of the chords as you approach P. You should be able to 'guess' that the tangent of $y = x^2$ at the point P(2.5, 6.25) is ……   Do the same in order to complete the following table:

P	Gradient of tangent
(2, 4)	
(2.5, 6.25)	
(3, 9)	
(3.5,)	
(4,)	

From the table on the left, you should be able to conjecture that for the curve $y = x^2$, the gradient of the tangent at any point on the curve is …… times the value of the x-coordinate of that point and that this holds for ANY point on the curve.

Now that you have a basic feel for the difference between chords and tangents and their respective gradients, this is a good place to finish with the applet and introduce some important notation. For this, you will have to download the "Differentiation 1 – From Chords to Tangents" theory sheet available from the same ***MathinSite*** web page from which this tutorial document was downloaded.

Generalisation from the curve $y = x^2$ to other curves is covered in other applets on ***MathinSite***.