Name ………………………………………   Course ……………………………….
Print and use this sheet with ***MathinSite***'s 'Exponential Functions' applet to have a permanent record of your work.

Exponentially changing Sinusoids using $y = A + Be^{kt}\sin wt$.

The Software Tutorial.

Learning Aims

- To develop an understanding of how the exponential function modifies a sinusoid.
- To develop, through experimentation, a feel for the effect of changing its parameters.
- To enable questions to be answered of a "what if ...?" nature.

Introduction

The multiplication of a sinusoid by an exponential function has important applications in the mathematical analysis of certain topics in, amongst others, science and technology. For example, under certain conditions the vibrations of a mass / spring / damper exhibits oscillations that eventually die away - the displacement of the mass about some datum position can be modelled using the given equation.

This package allows the user to investigate a generalised form of exponentially changing sinusoids using,

$$y = A + Be^{kt}\sin wt$$

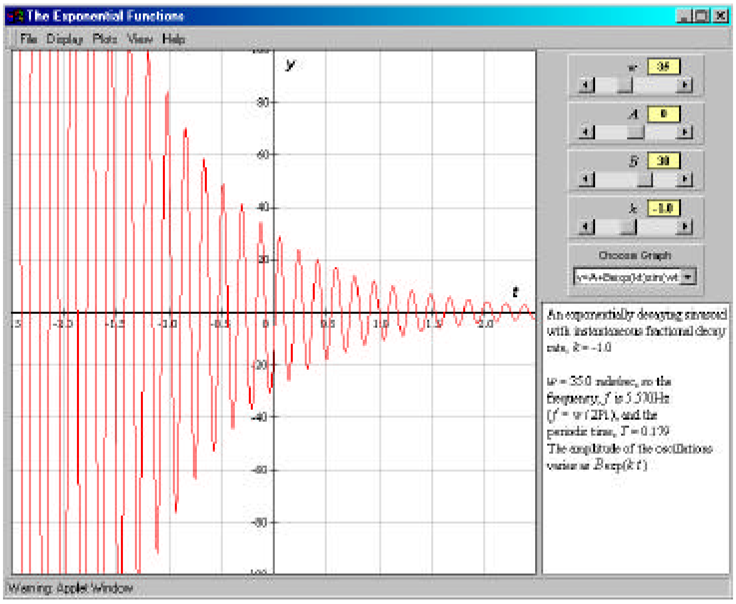
or, as is sometimes written,

$$y = A + B\exp(kt)\sin w\,t$$

This latter format is used in the software, and occasionally in written texts, for typographical reasons. The more commonly used format is $y = A + Be^{kt}\sin wt$. In this tutorial sheet you will be exposed to both formats to ensure familiarity with the use of e^{kt} and $\exp(kt)$.

Loading the Software

The applet, "Exponential Functions" is obtained from the web site from which this work sheet was downloaded. Choose $y = A + B\exp(kt)\sin w\,t$ from the 'drop-down' box under 'Choose Graph' below the three slider bars. You should now see an applet window similar to that shown here – notice a fourth slider, for w, becomes available.



Running the applet

The software uses the default values $A = 0$, $B = 30$, $k = -1.0$ and $w = 35$.

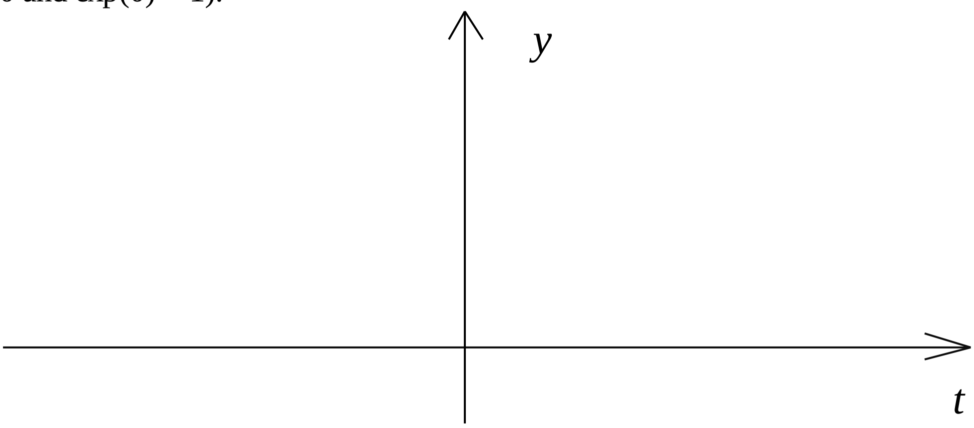
The four slider bars on the right of the window allow the user to change each of the above parameters. Clicking the arrow boxes at either end of the slider bar effect *small changes* in the parameter values. Clicking in the main part of the box of the slider bar makes *larger* *changes. Clicking, holding and moving the* <u>*slider*</u> *in the slider bar can make* ***any*** *sort of change.*

Changing the values by moving the sliders is the only means of user-input.

The software gives you the option of showing single plots or multiple plots (under 'Plots' in the pull down menu structure at the top of the applet window). It also allows you to 'drag' the plot around (to see areas not covered by the original window on the plot) by holding down the mouse button whilst at the same time moving the mouse over the plot. The plot can be recentred at any time using the 'Centre on Origin' option, under 'Display'. You can reset the software at any time to the default values (those used when the program loads – given above) using 'Reset' from the 'Plots' menu.

The Tutorial

Changing k The plot displayed shows $y = A + Be^{kt}\sin w\, t$ with default values $w = 35$, $A = 0$, $B = 30$ and $k = -1$. Change w to 15 by clicking twice to the left of the slider in the w slider bar. From the "Plot" menu at the top of the screen choose "Multiple Plots" and superimpose, by clicking twice to the *left* of the slider in the k slider bar, the two curves for $k = -2$ and -3. Sketch the three curves on the axes below, labelling each and indicating on the graph the value the y-intercept, i.e. where the curve crosses the y-axis (at that point $t = 0$ and $\exp(0) = 1$).



Write down the equations in full for the three curves you have just sketched.

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Describe the overall shape of the curve(s) and, in particular, the effect of changing the value of k on the 'oscillations'. Indicate whether the frequency of the oscillations is affected by changing the value of k. The above curves are all examples of exponentially decreasing sinusoids. (The official name for k is "the instantaneous fractional growth rate" and the effect of changing k is dealt with in some detail in the tutorial for $y = A + Be^{kt}$.)

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These curves all pass through the same point. What is it and why do you think it does this?
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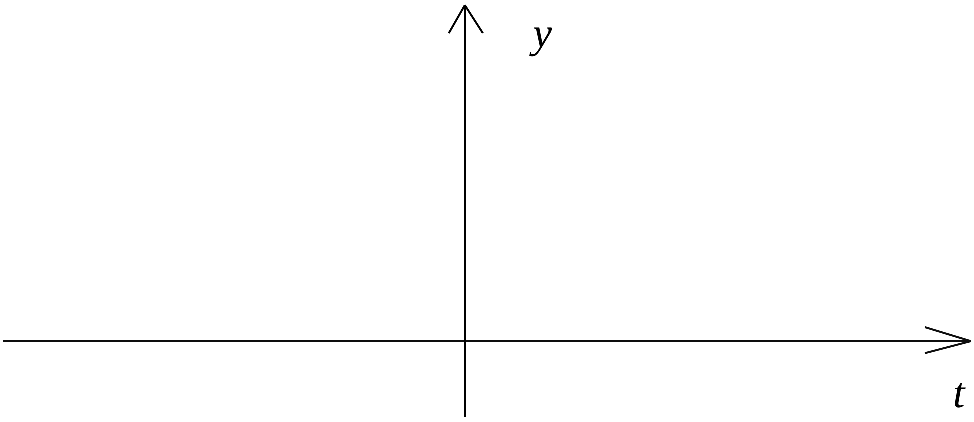
Now determine the effect of making k positive. If you have already completed the tutorial on y = A + Be^{kt} you might guess first what the effect will be. Leave w (at 15), A and B the same and try k = 1.0. Select 'Clear Multiple Plots' from the "Plots" menu. Superimpose the cases when k = 2 and 3. Sketch and label the curves below:



Describe the overall shape of the curves and, in particular, the effect of changing the value of k from k = 1.0 to k = 3.0. These are all examples of exponentially growing sinusoids.

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Describe what you notice about $y = 30\exp(-1.0t)\sin(15t)$ and $y = 30\exp(1.0t)\sin(15t)$, or, similarly, $y = 30\exp(-2.0t)\sin(15t)$ and $y = 30\exp(2.0t)\sin(15t)$, etc. Use the software to plot them together, if necessary.

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Did you notice that you didn't try k = 0? Before you do, guess what the curve will look like. Bear in mind that k represents a growth/decay rate, so what happens if k = 0? Plot it and see. Would you describe the curve as growth or decay? Why?

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Changing *B* Use the slider bars to set up w = 15, A = 0, B = 10 and k = 1.5 and select "Clear Multiple Plots". Now superimpose the curves for which B = 20, 30, 40 … as far as you need to sketch the family of curves obtained on the following page and to answer the questions below. Note that the curves all exhibit exponential growth since k is positive.

Describe the effect of increasing the value of B.

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What is the difference between changing k and changing B? (You may need to play with both k and B sliders to fully appreciate the difference.)
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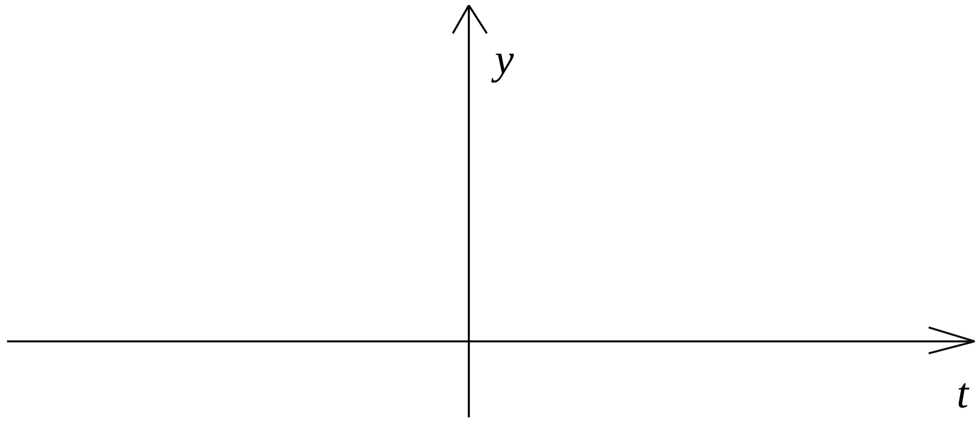
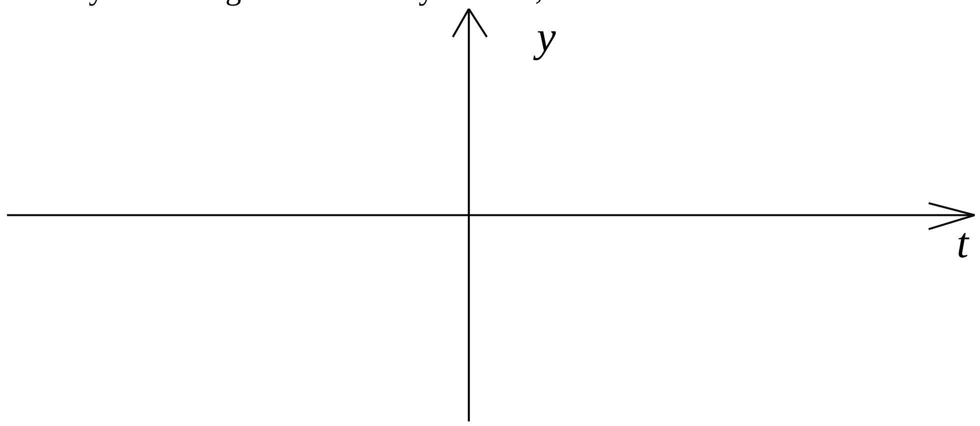


Now try B with negative values. Try B = -10, -20. Sketch and label the curves below.



What do you notice about the difference between the curves $y = 10\exp(1.5t)\sin(15t)$ and $y = -10\exp(-1.5t)\sin(15t)$, or, similarly, $y = 20\exp(1.5t)\sin(15t)$ and $y = -20\exp(-1.5t)\sin(15t)$, etc.?

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Did you notice that we didn't try $B = 0$? Before you do, guess what the curve will look like. Bear in mind that that this time, k is not zero - so what happens if $B = 0$ (and $A = 0$)? Plot it and see. Does the curve exhibit growth or decay? Why? What about the oscillations?

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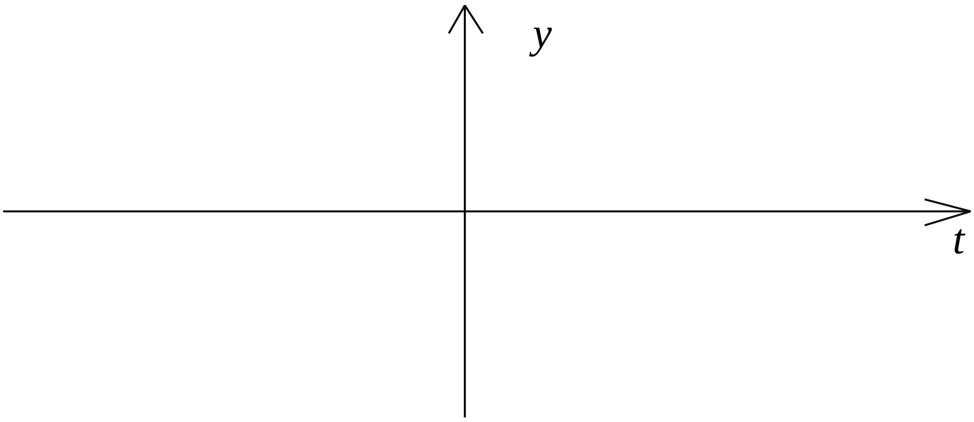
<u>Changing</u> w Now use the slider bars to set up $w = 10$, $A = 0$, $B = 80$ and $k = -1.5$ and select "Clear Multiple Plots". Superimpose the curves for which w = 20, 30 and 40. Sketch and label the curves on the axes on the next page.



Describe the effect of *increasing* the value of w and any effect that this has on the rate of decay. Describe how the plots help you in answering this.

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Why do you suppose that the software does not allow negative values of w ?

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Describe the effect on the graph of $y = Be^{kt}\sin w\,t$ of changing w .

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Describe the effect on the graph of $y = Be^{kt}\sin w\,t$ of changing B.

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Describe the effect on the graph of $y = Be^{kt}\sin w\,t$ of changing k.

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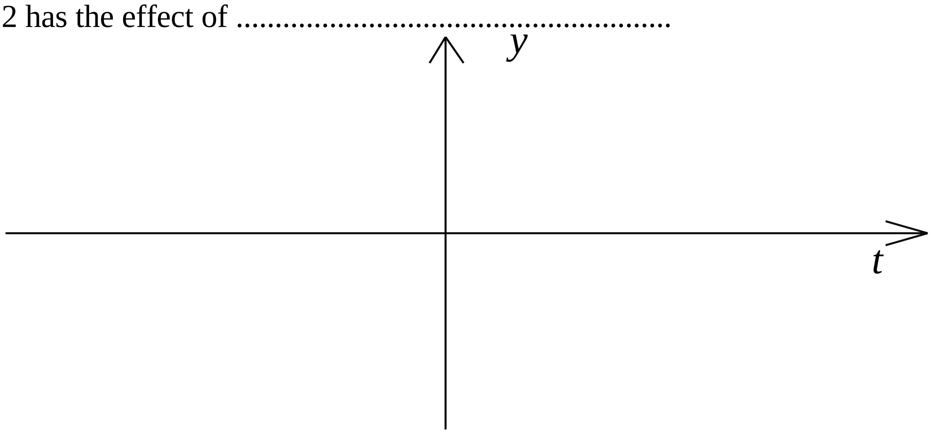
Try this without using the software. Sketch the graph of $y = 20e^{-3t}\sin 2t$. (Hint: e^{-3t} represents exponential .... , the 20 has the effect of .... , the 2 has the effect of ....



Use the software to check your answer. Do the same for the following:

$y = 25\exp(3t)\sin 3t$ $\quad$ $y = 25e^{3t}\sin 15t$

$y = 25\exp(-3t)\sin 3t$ $\quad$ $y = -10e^{5t}\sin t$ (What happened to the oscillations?)

$y = -55e^{-t}\sin 35t$ $\quad$ $y = -40\exp(-2t)\sin(100t)$

Try this – it gives an interesting result. Use the software to clear the screen and adjust the sliders to show the graph of y = 40exp(-2t)sin(50t). Select "Multiple Plots" and then select y = A + Bexp(kt) from the box on the right of the screen. The slider bar for w should disappear but the others should still show k = -2, B = 40 and A = 0. Use the plots to describe how the curve $y = B\exp(kt)$ relates to the curve $y = B\exp(kt)\sin(w\,t)$. (A = 0)

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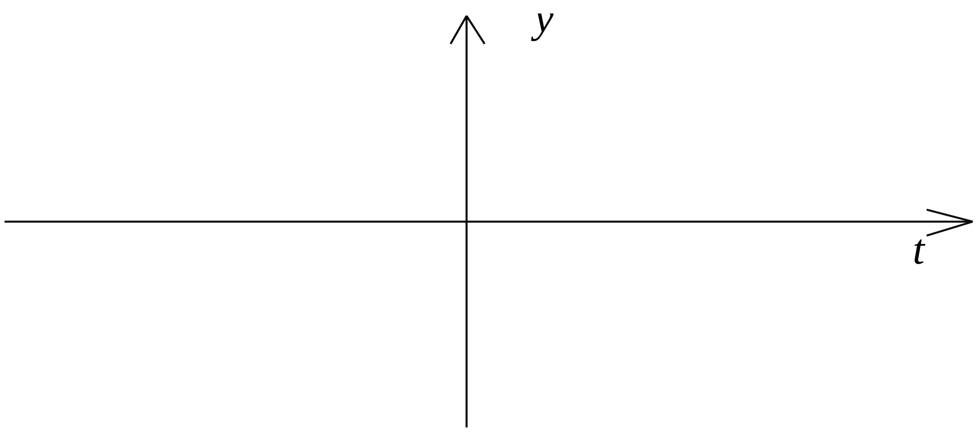
Sketch and label the two curves on the following axes.



<u>Changing *A*</u>

If you have already tried the Tutorial Sheet for $y = A + Be^{kt}$ you will know the answer to:

What is the effect on the graph of ***in***creasing A?

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What is the effect on the graph of ***de***creasing A?

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To see the effect of this, use the slider bars to set up w = 10, A = 0, B = 30 and k = -1 and select "Single Plot" from the "Plots" menu. Click and hold down the mouse cursor on the arrow on the ***right***-hand end of the A slider. The *value* of A increases in the 'A' number box - and the graph ……………………… as expected!

Now click and hold down the mouse cursor on the arrow on the ***left***-hand end of the A slider. The value of A decreases in the 'A' number box - and the graph ………………… as expected!

You may like to repeat this exercise using the "Multiple Plots" option.