

The Exponential Function $y = B\text{exp}(kt^2)$

The Software Tutorial.



Learning Aims

- To develop an understanding of the exponential function $y = B\text{exp}(kt^2)$.
- To develop, through experimentation, a feel for the effect of changing its parameters.
- To enable questions to be answered of a "what if ...?" nature.

Introduction

The exponential function $y = Be^{kt^2}$ has important applications in Statistics. For certain values of B and k it is the equation of the curve that represents the normal distribution. The "Standard Normal Curve" used statistical analysis has a probability density function:

$$f(x) = \frac{1}{s\sqrt{2p}} e^{-\frac{1}{2}\left(\frac{x-m}{s}\right)^2}$$

You can see that these two expressions are the same, with

$$B = \frac{1}{s\sqrt{2p}}, \quad k = -\frac{1}{2} \quad \text{and} \quad t = \frac{x-m}{s}$$

where m and s are the mean and standard deviation of the distribution.

This package allows the user to investigate the standard normal curve equation using,

$$y = Be^{kt^2}$$

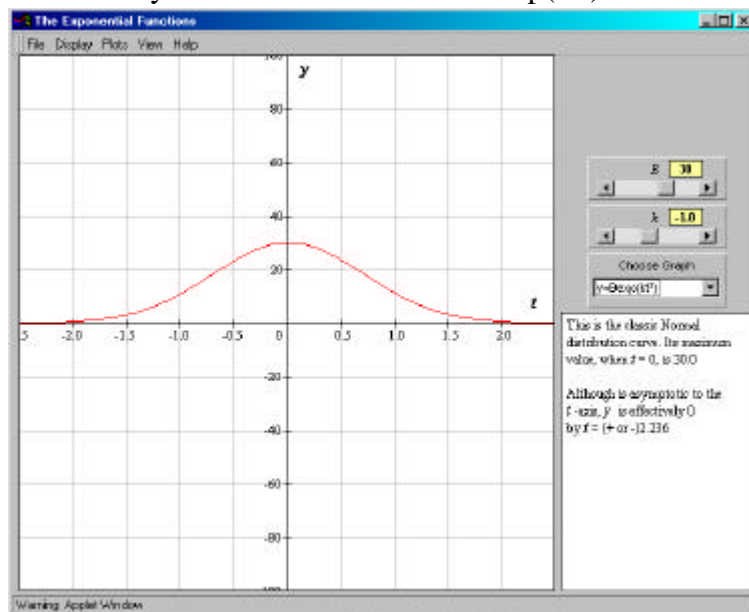
or, as is written in the software, $y = B\text{exp}(kt^2)$

For typographical reasons, this latter format is used in the software, and occasionally in written texts. The more commonly used format is $y = Be^{kt^2}$. In this tutorial sheet you will be exposed to both formats to ensure familiarity with the use of Be^{kt^2} and $B\text{exp}(kt^2)$.

Loading the Software

The applet, "Exponential Functions" is obtained from the web site from which this work sheet was downloaded. Choose

$y = B\text{exp}(kt^2)$ from the 'drop-down' box under 'Choose Graph' below the three slider bars. You should now see an applet window similar to that shown here – notice that the slider for A is not needed in this case, so becomes unavailable.



Running the applet

The software uses the default values $B = 30$ and $k = -1.0$

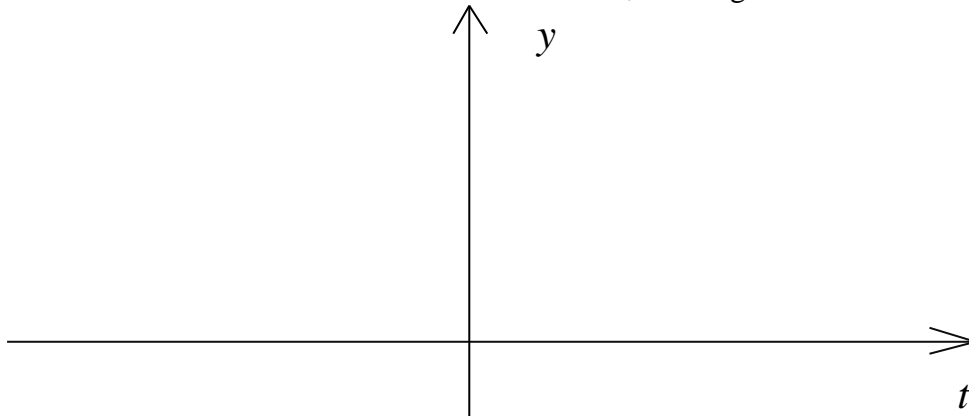
These two slider bars on the right of the applet window allow the user to change each of the above parameters. Clicking the arrow boxes at either end of the slider bar effect *small changes* in the parameter values. Clicking in the main part of the box of the slider bar makes *larger changes*. *Clicking, holding and moving the slider in the slider bar can make any sort of change.*

Changing the values by moving the sliders is the only means of user-input.

The software gives you the option of showing single plots or multiple plots (under 'Plots' in the pull down menu structure at the top of the applet window). It also allows you to 'drag' the plot around (to see areas not covered by the original window on the plot) by holding down the mouse button whilst at the same time moving the mouse over the plot. The plot can be recentred at any time using the 'Centre on Origin' option, under 'Display'. You can reset the software at any time to the default values (those used when the program loads – given above) using 'Reset' from the 'Plots' menu.

The Tutorial

Changing k Use the slider bars to change the values of B and k to $B = 90$ and $k = -1$. From the "Plot" menu at the top of the screen choose "Multiple Plots". Superimpose, by clicking to the left of the slider in the k slider bar, the four curves for $B = 90$ and $k = -2, -3, -4$ and -5 . Sketch the five curves on the axes below, labelling each.



Write down here the equations in full for the five curves you have just sketched.

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Describe the overall shape of the curves and, in particular, the effect of changing the value of k . Describe how the maximum value of the curve is affected by changing k .

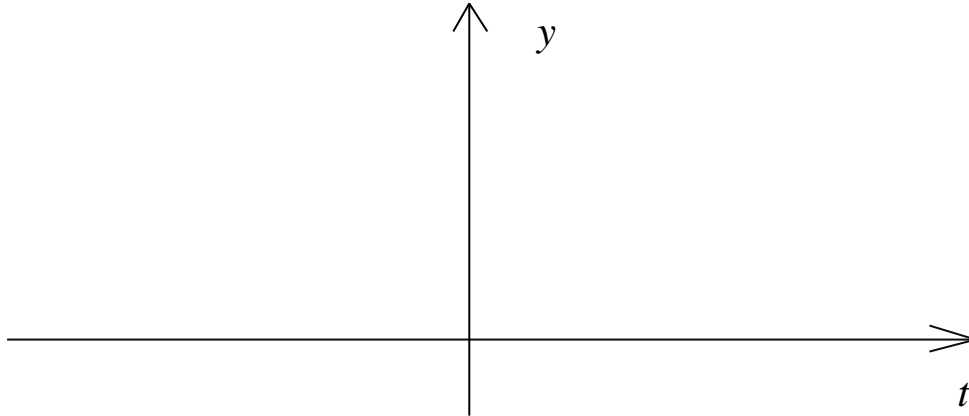
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These curves all pass through the same point. What is it and why do you think it does this?

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Now determine the effect of making k positive. Choose $B = 30$ and try $k = 1.0$ and press "Clear Multiple Plots". Superimpose the cases when $k = 2.0, 3.0$ and 4.0 ('drag' the graph down by clicking and holding down the mouse button in the graphics area and 'pulling' the graph down if you wish to see what happens to the graph outside the window displayed.) Sketch and label the curves below:



Describe the overall shape of the curve(s) and, in particular, the effect of changing the value of k from $k = 1$ to 5 . Do these curves represent a normal distribution curve?

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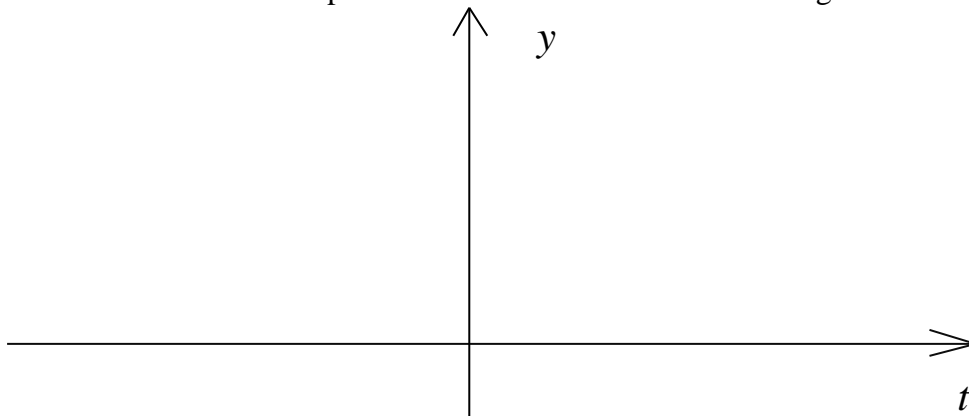
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Did you notice that we didn't try $k = 0$? Before you do, guess what the curve will look like. Plot it and see. Would you describe the curve as normally distributed? Why/why not?

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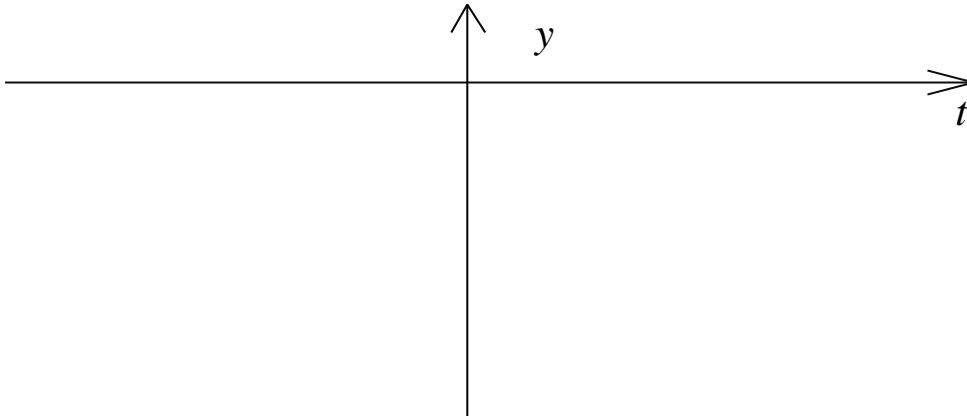
Changing B Use the slider bars to set up $B = 10$ and $k = -1.5$, clear the screen and plot this curve. Move the graphics area if necessary, or choose "Centre on Origin" from the "Display" menu. Now superimpose the curves for which $B = 20, 30, 40, \dots$ as far as you need to sketch the family of curves obtained and to answer the following question. Note that the curves are all examples of a normal distribution since k is negative and B is positive.



Describe the effect of increasing the value of B , including the effect on the 'width'.

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Now consider B with negative values. Try $B = -10$, $B = -20$, $B = -30$, $B = -40$, Sketch and label the curves below.



What do you notice about the difference between the curves $y = 10\exp(1.5t^2)$ and $y = -10\exp(1.5t^2)$, or, similarly, $y = 20\exp(1.5t^2)$ and $y = -20\exp(1.5t^2)$, etc.?

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Did you notice that we didn't try $B = 0$? Before you do, guess what the curve will look like. Bear in mind that that this time k is not zero, so what happens if $B = 0$? Plot it and see. Would you describe the curve as normally distributed or not? Why?

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Describe the effect on the graph of $y = B\exp(kt^2)$ of changing the **sign** of B .

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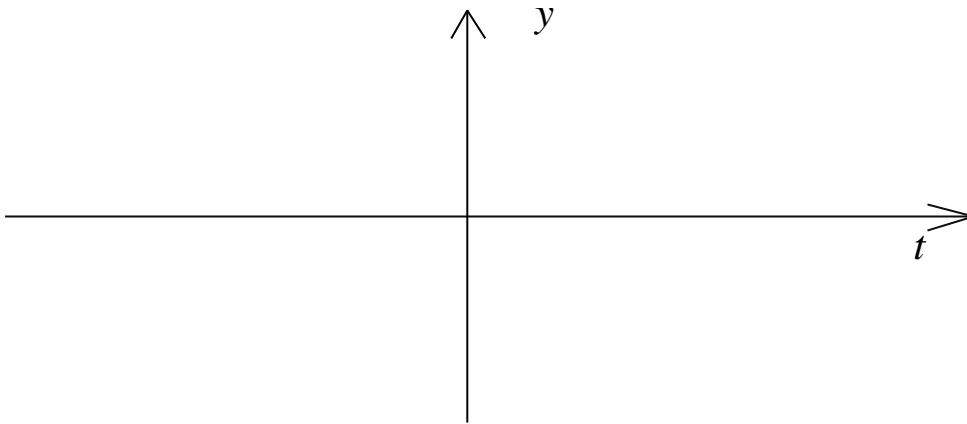
Describe the effect on the graph of $y = B\exp(kt^2)$ of changing B with $k < 0$.

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Describe the effect on the graph of $y = B\exp(kt^2)$ of changing k (with $k < 0$).

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Try this without using the software. Sketch the graph of $y = 20\exp(-2t^2)$ on the axes on the next page. Label the curve and mark on the graph any important value(s).



Use the software to check your answer.

Do the same for the following:

$$y = 25\exp(3t^2)$$

$$y = 75\exp(-3t^2)$$

$$y = 10 + 5e^{-t^2}$$

$$y = 25\exp(-3t^2)$$

$$y = -10\exp 3t^2$$

$$y = -40\exp(-2t^2)$$