Name ……………………………………… Course ………………………………..

Print and use this sheet in conjunction with ***MathinSite***'s 'LR Series Circuit' applet and theory sheet.

The LR Series Circuit - Theory Sheet 2

The Sinusoidal Response of an LR Circuit which has an applied sinusoidal voltage, *e*(*t*) = *a* sin*w t*

When a sinusoidal voltage $e(t) = a \sin w t$ is applied to the LR circuit, a sinusoidal steady state current occurs. Sometimes this will be of the form $i(t) = A \sin w t$. More generally, however, it will take the form $i(t) = A \sin w t + B \cos w t$. This latter format does not lend itself to close investigation of the steady state response. However, expressing $A \sin wt + B \cos wt$ as $R \sin(wt + f)$ does.

Example: The default circuit with the *default* applied sinusoidal voltage.

With default input $e(t) = 20 \sin 400t$, the default circuit response is given by the current $i(t) = 5e^{-400t} - \cos(400t) + \sin(400t)$. The exponential term is the transient response and the sinusoidal terms are the steady state response.

Consider just the steady state part of the solution,

$$-\cos 400t + \sin 400t \qquad \text{(i)}$$

This can be expressed in the form

$$R \sin(wt + f),$$

$$\boxed{\text{where } R\sin(wt + f) = R\sin wt \cos f + R \cos wt \sin f}$$

(*For further work on trigonometrical identities such as this, see your mathematics textbooks.*)

In the case here, then

$$R \sin(400t + f) = R \sin 400t \cos f + R \cos 400t \sin f \qquad \text{(ii)}$$

Comparing (i) and (ii) gives

$$R \sin f = -1 \quad \text{and} \quad R \cos f = +1$$

Squaring and adding these gives

$$R^2 \sin^2 f + R^2 \cos^2 f = (-1)^2 + (+1)^2$$

$$\text{so } \underline{R = +\sqrt{2}} \qquad (\text{since } \sin^2 f + \cos^2 f = 1)$$

Dividing gives

$$\frac{R \sin f}{R \cos f} = \frac{-1}{+1} \quad \text{or} \quad \tan f = -1,$$

$$\therefore f = \tan^{-1}(-1) = -\frac{p}{4} \text{ or } \frac{3p}{4}$$

However, since R is positive, and $R \sin f < 0$ and $R \cos f > 0$,

f must be in the 4th quadrant

$$\text{so } \underline{f = -\frac{p}{4}},$$

so $\underline{-\cos 400t + \sin 400t}$ can be rewritten as $\underline{\sqrt{2} \sin(400t - p/4)}$

$\therefore$ input is $20 \sin 400t$ and the steady state output is $\sqrt{2} \sin(400t - p/4)$

This result is summarised in the table below, but of particular note is that the steady state sinusoid has the same frequency ($\omega/2\pi$) as the driving voltage but it is out of phase with the driving voltage by $-\pi/4$ $(= -2\pi/8)$ i.e. by one-eighth of a cycle. The 'minus' indicates that the steady state current ***lags*** the input voltage. This lag can be calculated in seconds by considering

$$400t - \pi/4 = 0 \qquad \text{i.e. } t = \pi/1600 \approx 1.96 \times 10^{-3}$$

So steady state output lags input by about 2 ms. Now the transient decays after about 12.5 ms and using the mouse pointer (with "Show mouse co-ords" on, from the Display menu) it can be seen that the steady state rises across the time axis at about 17.65 milliseconds. (Try it!) In the text area, the periodic time is given as 15.708 ms. Subtraction of these values shows that the steady state must 'begin' a complete cycle at about +2 ms. This can be 'seen' graphically if you project the steady state sinusoid *back* through the transient period.

So for *this* case,

	Input $e(t) = 20\sin 400t$	**Steady State output** $i(t) = \sqrt{2}\sin\left(400t - \pi/4\right)$
amplitude	20	$\sqrt{2}$
angular velocity	400	400
frequency	$400/2\pi$	$400/2\pi$
phase shift (in radians)	Zero (datum)	$-\dfrac{\pi}{4}$

… and in general,

	Input $e(t) = a\sin\omega t$	**Steady State output** $i(t) = A\sin\omega t + B\cos\omega t$
amplitude	a	$\sqrt{A^2 + B^2}$
angular velocity	ω	ω
frequency	$\omega/2\pi$	$\omega/2\pi$
periodic time	$2\pi/\omega$	$2\pi/\omega$
phase shift (in radians)	Zero (datum)	$\tan^{-1}\left(\dfrac{B}{A}\right)$

So note that an LR series circuit changes the amplitudes between input and output and may produce a phase shift – but the angular velocity, ω, (and hence frequency and periodic time) is the same for both input and output.

NOTE $\tan^{-1}\left(\dfrac{B}{A}\right)$ must be in the correct quadrant according to

	A	B	$\phi = \tan^{-1}(B/A)$
$A\sin\omega t + B\cos\omega t$ = $R\sin(\omega t + \phi)$	+	+	1st quadrant
	-	+	2nd quadrant
	-	-	3rd quadrant
	+	-	4th quadrant