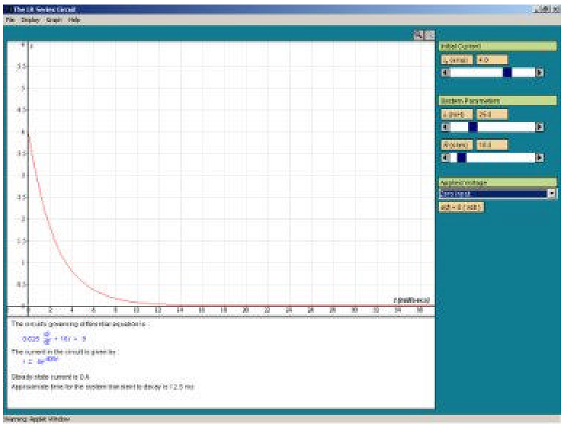
The LR Series Circuit
Tutorial Sheet 1
- for circuits with no applied voltage

The **Learning Outcomes**, **Background** information and the **Prerequisites** are discussed on the associated **LR Series Circuit Theory Sheet**, also available from ***MathinSite*** on http://mathinsite.bmth.ac.uk/html/applets.html.

Loading the LR Series Circuit Applet

Loading the applet from ***MathinSite***, will produce a display similar to that shown here.



Running the applet

The software loads with the default values:

- i_o = 4 A
- L = 25 mH (note that the value from the L scrollbar is automatically in mH)
- $R = 10\ \Omega$.
- Zero input (i.e. no applied voltage).

With these default values the graphical response (resulting current in the circuit) is displayed in the main graphics area and the relevant mathematical information is shown in the text area. *Note: the default values can be reinstated at any time using the 'Reset' option from the 'Graph' menu.*

The slider bars on the upper right of the applet window allow the user to change the value of various circuit parameters. Clicking the arrows at either end of the slider bar effect *small changes* in the parameter value. Clicking in the main part of the box of the slider bar makes *larger changes*. Clicking, holding and moving the slider in the slider bar can make any sort of change.

Changing the parameter values by moving sliders and choosing the type of applied voltage from the drop-down box are the only user-input required.

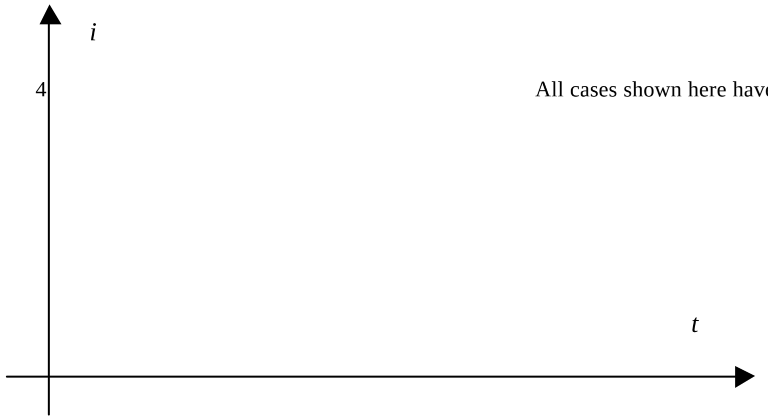
The applet gives you the option of showing single or multiple plots (under the 'Graph' menu at the top of the applet window). It also allows you to 'drag' the plot (to see areas not covered by the original window on the plot) by holding the mouse button whilst at the same time moving the mouse over the plot. The applet's default values can be reset at any time using the 'Reset graph values' option, also under 'Graph'.

Reminder: *The applet may appear in a less-than-full-size window. To give you the best-sized image, use the 'maximise' button, the middle button in the top right-hand corner of the window, to fill your screen with the LR Series Circuit applet window.*

The Tutorial

Changing *R*

Sketch on the following axes the graph of the response of the default system (L = 25 mH, R = 10 Ω, i_0 = 4 and zero applied voltage). Indicate R = 10 against this graph. Indicate a scale along the i and t axes.



Circle below the type of mathematical function that describes this response.

Linear / Quadratic / Cubic / Exponential / Sinusoidal (Circle one)

The approximate time taken for the current to become zero here is ……… ms.
This, the ***transient time*** (time taken for any transient effects to decay), is related to 'five-times-the-time-constant' (see "Note on Time Constants", theory sheet).

Now use the R slider to change R in turn to R = 9, 8, 7, 6, 5. (You may like to use the 'Change to Multiple Plots' option in the 'Graph' menu to see the effect more clearly.) The multiple plots you sketch above should clearly show the effect of reducing R. At the same time, complete the missing entries in the following table:

R (Ω)	L (H)	i_0	Current, i, at t seconds	Time Constant, t (s)	Transient decay Time (ms)
10	25×10^{-3}	4	$i = 4e^{-400t}$	$\frac{1}{400} = 0.0025$	12.5
9	25×10^{-3}				
	25×10^{-3}				
	25×10^{-3}	4			
	25×10^{-3}				
5	25×10^{-3}				

From the graphs, what is the 'steady state' (i.e. eventual) current in each case?

....

What is the input to this circuit in all cases?

....

What was the phrase used (in the Theory Sheet) to indicate that a system is 'linear'?

....

Briefly describe here the effect on the response (current) when the *R* is reduced.

……………………………………………………………………………………

Describe why you think this happens (you may like to ask your Physics or Electronics lecturer/teacher for the answer).

……………………………………………………………………………………

What is the *steady state* value of the resulting current in all cases here (i.e. to what value does the resulting current settle down in the 'long term')?

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Why do you think it settles down to this particular value?

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What is the relationship between the values in the last and next-to-last columns?

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How is the transient time affected by reducing the resistance?

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The transient time is approximately $5 \times t = \frac{5 \times L}{R}$. How does this mathematical statement corroborate your previous answer?

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In anticipation of the next section and from the information in the previous question, how do you think the transient time will be affected as *L* decreases?

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Changing *L*

From the 'Graph' menu, choose 'Reset all graph values'. Sketch on the following axes the graph of the response of the default system again (i.e. L = 25 mH, R = 10 Ω, i_0 = 4 and zero applied voltage). Indicate L = 25 mH against this graph. Indicate a scale along the i and t axes.

All cases shown here have R = 10 and $e(t) = 0$

Now use the L slider to change L in turn to R = 20, 15, 10, 5. (Click between the left of the L slider and the left-facing arrow.) The multiple plots you sketch above should clearly show the effect of reducing L. At the same time, complete the missing entries in the following table:

R (Ω)	L (mH)	i_0	Current, i, at t seconds	Time Constant, t (s)	$5\times t$ (ms)
10	25	4	$i = 4e^{-400t}$	$\frac{1}{400} = 0.0025$	12.5
10	20				
10					
10					
10	10				
10					

Briefly describe here the effect on the response (current) when L is reduced.

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Describe why you think this happens (you may like to ask your Physics or Electronics lecturer/teacher for the answer).

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How is the transient time affected by reducing the inductance?

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Changing i_0

If, initially, you have a larger current in the circuit, do you think the current will take longer or shorter to decay, or will there be no change in the transient response time?

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From the 'Graph' menu, choose 'Reset all graph values'. Sketch the graph of the response of the default system yet again (i.e. L = 25 mH, R = 10 Ω, i_0 = 4 and zero applied voltage). Indicate i_0 = 4 against this graph. Indicate a scale along the i and t axes.

Now use the i_0 slider to change i_0 in turn to i_0 = 6, 8 and 10. The multiple plots you sketch above should clearly show the effect of reducing L. At the same time, complete the missing entries in the following table:

i_0 (A)	L (mH)	R (Ω)	Transient decay time (ms)
4	25	10	12.5
6	25	10	
8	25	10	
10	25	10	

Briefly describe here the total effect on the response (current) when the i_0 is changed.

………………………………………………………………………………………

… and how was the transient time affected by changing the initial current?

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Briefly describe here the total effect on the response (current) when the i_0 is *negative*. (You may need to use the mouse to 'grab and move' the graph to see it in places where the response moves out of the graphics area.)

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What does a *negative* i_0 mean in physical terms?

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Put i_0 = 0. Explain the response you see in physical terms.

Summary

- **Changing *L* and/or *R* will not affect the *overall* shape of the response curve (it will always be exponential decay). There are some exceptions when this is not, or does not appear to be the case. For example, zero initial current and zero applied voltage result in a zero (flat) response and, for a negative i_0 (when the current flows in the negative direction), the response is "exponential growth to a limit" (see also *MathinSite*'s Exponential Functions applet).**

- **However, changing *L* and/or *R will* affect how quickly the transient response (the exponential decay) lasts – either increasing *L* or decreasing *R* will make the transient last longer giving a slow transient response. The circuit will have a *faster* transient response if *L* is reduced and/or *R* is increased. The transient response time will be unaffected if *L* and *R* are together increased (or decreased) in the same ratio.**

- **The initial response is 'short lived', so is called the *transient* response.**

- **Time taken for the initial, transient response to die away is given by $5t$ (approx.)**

- **NOTE: some authors use $6t$ for the transient time.**

- **Changing i_0 has the effect of changing the point from which the response curve 'starts' (when $t = 0$). It modifies the overall response curve (while remaining exponential) but does not affect the transient time or how the circuit response settles once the transient has died away.**

- **A circuit that has no initial current flowing in it and has no external voltage driving it *necessarily* has zero response for all time.**

Exercises

1.

(a) Use the applet's scrollbars (enabling you to change the initial condition and system parameters) to design a zero input LR series circuit that initially has a current of 3A, a resistance of 10 ohm and a transient (settling) time of approximately 25 milliseconds. Complete the ***first row*** of the following table.

Initial value	*L*	*R*	Differential equation	Solution	*L/R*
3		10		$i =$	
3		5		$i =$	
3	100			$i =$	

(b) This is not the only LR circuit that settles in 25 ms. Change *R* and *L* as necessary to produce circuits with a transient time of 25 ms for the cases shown in the second and third rows of the above table. Complete the table.

(c) In what way(s) do these 2nd and 3rd solutions above differ from the first?

(d) What can you say about the value of L/R in all three cases? Why is this what you might have expected? (The clue is in part (b).)

(e) What do you notice about the differential equations relating to each of these circuits?

(f) What were the currents at $t = 0$ in each case? Why?

2.

a) Adjust the sliders of the applet to design a circuit that has an initial current of 3.5 amp and a transient settling time of 15 ms.

b) Sketch the response curve indicating any important quantities on your graph.

c) Now choose two other values of *L* and *R* to produce *any other* circuit with the same properties.

d) Using the definition of settling time ($5\times t$) given above, why is it possible to design two different zero-driven LR circuits with the same response?

e) Use Laplace Transforms or Integration to solve the governing differential equation for either case.