Name ………………………………………                                    Course ……………………………….

Print and use this sheet in conjunction with ***MathinSite***'s 'LR Series Circuit' applet and theory sheet.

# The LR Series Circuit
# Tutorial Sheet 2
# – circuits with applied ramp voltages

**Learning Outcomes**, **Background** information, **Prerequisites** and how to load and run the accompanying applet are all dealt with either in the LR Series Circuit Theory Sheet or Tutorial Sheet 1.

NOTE: a **ramp function** is a linear function of the form $f(t) = bt + c$. Either of the $b$ or $c$ could be zero.

## The Tutorial

If you have been using the applet to complete Tutorial Sheet 1, you may need to reset the default values ($L$ = 25 mH, $R$ = 10 Ω, $i_0$ = 4 and zero applied voltage). These can be reset at any time by choosing 'Reset all graph values' from the 'Graph' menu.

## Changing the Applied Voltage

### (1) Constant Voltages

Sketch on the following axes the graph of the response of the default system. Indicate that '$e(t) = 0$' against this graph. Indicate a scale along the $i$ and $t$ axes.


The approximate time taken for the current to become zero, the ***transient time***, is ……… ms.

Now choose "Algebraic" from the drop-down box below the $L$ and $R$ scrollbars. Doing this applies a voltage to the LR circuit that has the form, $e(t) = bt + c$. In the default case, the circuit is subjected to the constant applied voltage, $e(t) = 20$. *Note that the response is still exponential decay*.

Now click on the arrow at the right-hand end of the $c$ slider to change $c$ successively to 30, 40, 50, 60. (You may like to use the 'Change to Multiple Plots' option in the 'Graph' menu to see the effect more clearly.) *Note that the resulting current eventually settles down to the* ***steady state value*** *- here, a constant value in each case*.

Sketch these multiple plots above to clearly show the effect of changing $c$, marking against each graph, '$e(t) = 20$', etc. At the same time, complete the following table:

| $i_0$ (A) | $L$ (mH) | $R$ ($\Omega$) | $c$ (V) | Differential Equation | Steady state value (A) | Transient decay time (ms) |
|---|---|---|---|---|---|---|
| 4 | 25 | 10 | 0 | $0.025\frac{di}{dt}+10i=0$ | 0 | 12.5 |
| 4 | 25 | 10 | 20 | | | |
| 4 | 25 | 10 | 30 | | | |
| 4 | 25 | 10 | 40 | | | |
| 4 | 25 | 10 | 50 | | | |
| 4 | 25 | 10 | 60 | | | |

Briefly describe here the effect on the steady state current when $c$ is changed.

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Give a brief physical description of why you think this happens.

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When the current in the circuit isn't changing (i.e. $di/dt = 0$), the potential drop across the inductor is zero. Write the resulting general governing 'differential' equation here in terms of $R$, $i$ and $c$. Rearrange this equation to make $i$ the subject.

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Briefly explain the relevance of your answer to the previous question in terms of the contents of the $R$, $c$ and 'steady state value' columns in the table above.

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Explain why the case $e(t) = 40$ (volts) results in a horizontal straight line.

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What effect did changing $c$ above a value of 40 have on the resulting curve?

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(*This type of curve is no longer called exponential decay, but* ***exponential growth to a limit***.)

How is the transient time affected by changing $c$?

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Recall from Tutorial Sheet 1 that the time for the transient to decay is approximately $5\times t = \frac{5\times L}{R}$. How does this corroborate your previous answer?

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## (2) Ramp Voltages

Reset the default system using 'Reset all graph values' from the 'Graph' menu. Make sure the applied voltage, 'Algebraic', is showing in the drop-down box on the right. Sketch on the following axes the graph of the response of the default 'Algebraic' system. Indicate that the applied voltage is $e(t) = 20$ against this graph. Indicate a scale along the $i$ and $t$ axes.


Remember that "Algebraic" means that a linear voltage of the form $e(t) = bt + c$ is applied to the LR circuit. In the default case, as before, the circuit is subjected to the constant applied voltage, $e(t) = 20$.

Now click successively *between* the arrow at the right-hand end and the central slider of the $b$ scrollbar to change $b$ to 50, 100, 150, 200. (You may like to use the 'Change to Multiple Plots' option in the 'Graph' menu to see the effect more clearly.)

The steady state response of the circuit is no longer constant. If you need to, click and hold anywhere in the graph area and pull it to the left to see the steady state response (once the transient has died away) and complete the following statement:

An input *ramp* voltage results in an output …………… steady state current.

Superimpose the plots obtained on the above axes to clearly show the effect of changing $b$, marking against each graph, '$e(t) = 50t + 20$', etc. At the same time, complete the following table:

| $i_0$ (A) | $L$ (mH) | $R$ ($\Omega$) | $bt + c$ (V) | Differential Equation | Steady state (A) | Transient decay time (ms) |
|---|---|---|---|---|---|---|
| 4 | 25 | 10 | $0t + 20$ | $0.025\frac{di}{dt} + 10i = 20$ | $2 + 0t$ | 12.5 |
| 4 | 25 | 10 | $50t + 20$ | | $1.988 + 5t$ | |
| 4 | 25 | 10 | | | | |
| 4 | 25 | 10 | $150t + 20$ | | | |
| 4 | 25 | 10 | | | | |

For the mathematical solution shown above, explain why the $1.988 + 5t$ is the <u>steady state</u> response (remember: exponential decay).

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Note that *steady state* does not mean *constant <u>value</u>*. It means that the output settles down to a constant *<u>state</u>* with a mathematical solution that holds for all time – theoretically. Here, a steady ramp output.

Reminder from the Theory Sheet: The *solution* of the differential equation comes in two parts. For the case $L = 0.025$, $R = 10$ and $i_o = 4$ with applied voltage $50t + 20$, for example, these are $2.013e^{-400t}$ and $1.988 + 5t$. The $2.013e^{-400t}$ dictates the length of decay of the transient and *is based on the system <u>only</u>*, i.e. the values of $L$ and $R$. This part of the solution is called the ***Complementary Function*** (CF). The $1.988 + 5t$ *is based upon the type of input* (applied voltage) *<u>only</u>* and is called the ***Particular Integral*** (PI). This is generalised in the Appendix to the Theory Sheet.

If you haven't moved anything, your applet is set up for the case shown in the last line of the above table, i.e. the case when $bt + c = 200t + 20$. If it isn't, set it up for this case now but then change $c$ to the value zero.

So the governing differential equation is $0.025\frac{di}{dt} + 10i = 200t$, with $i_o = 4$.

If you project the steady state line back, where does it intersect the vertical axis? (You *can* obtain this graphically, but a more accurate answer can be obtained from the solution given by the applet.) ……………………………………………………

For this case, the equation of the *steady state* ramp current is $i =$ ………………

The time for the transient to decay is ……………………………………………

Now choose "Zero input" from the "Applied Voltage" drop-down box. Note carefully how the graphical output differs from the "Zero Input" case. (You may need to flip backwards and forwards between the two cases to fully appreciate the changes.)

For the zero input case, the equation of the steady state ramp current is $i =$ …………

… and the time for the transient to decay is …………………………………………

Describe the similarities and differences between these two cases (i.e. $i_o$, $L$ and $R$ unchanged but with the applied voltages $e(t) = 0$ and $e(t) = 200t$). You should include mention of how the transient is affected by the applied voltage.

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## Summary

With an applied ramp voltage, the LR circuit equation is $L\frac{di}{dt} + Ri = bt + c$

- **The solution (current) for the LR circuit with an applied ramp voltage is always of the form:**

$$i(t) = Ae^{-kt} + Bt + C$$

**It contains two parts:**

  - **The Complementary Function (CF), $Ae^{-kt}$, is the transient. $-kt$ depends on the circuit parameters $L$ and $R$ only, so dictating that the time for transient decay is dependent only on the circuit itself.**
  - **The Particular Integral (PI), $Bt + C$, a ramp output, is the steady state. *Note again, the type of steady state solution (long-term output) depends directly upon the type of applied voltage (input).***
- **Note these three cases, each indicating further how input dictates output:**
  - **If $b$ and $c$ are both zero (zero applied voltage), the steady state is zero**
    - **(*Zero input implies zero steady state output*)**
  - **If $b = 0$ and $c$ is not zero, the steady state solution is constant**
    - **(*Constant input implies constant steady state output*)**
  - **If $b$ is not zero, the steady state solution is a ramp**
    - **(*Ramp input implies ramp steady state output*)**
- **If in the PI (i.e. in the $Bt + C$ part of the solution only),**
  - **$C < i_o$, the transient exhibits *exponential decay* to the steady state**
  - **$C > i_o$, the transient is *exponential growth to a limit* to the steady state**
  - **$C = i_o$, there is no transient, the solution consists solely of the steady state**

## Exercise

a) Adjust the sliders of the applet to produce a circuit with, $L = 0.025$, $R = 10$, the applied voltage is 'Algebraic' with equation $e(t) = 400t + 50$, i.e. a ramp voltage, and an initial current of 1.9 amp.

b) Sketch the response curve indicating any important quantities on your graph.

c) Now change $c$ to 40 (so that now $e(t) = 400t + 40$) and superimpose this on your graph. (Using Multiple Plots may help here.)

d) Now repeat this for $c$ = 30, 20 and 10.

e) Comment on these five graphs with particular reference to the summary contents above.

f) For one case in (e), the transient term disappears. Use Laplace Transforms to solve this case to see mathematically what happens to the transient.

g) Explain physically why there is no transient.

h) For the special case mentioned in f), does the comment on the last line of the applet's text box still apply?

**Hint for f): If, after taking Laplace Transforms and multiplying through by 40 (to eliminate the 0.025), you obtain a factor $1.9s^2 + 800s + 16000$, take heart, this factorises "nicely"!**