The LR Series Circuit
Tutorial Sheet 3
-with applied sinusoidal voltages, $e(t) = a \sin \omega t$

Learning Outcomes, **Introduction**, **Loading the Software** and **Running the applet** have all been covered in the previous LR Series Circuit sheets.

NOTE: to better understand the workings of the LR Circuit with an applied sinusoidal voltage, you should know how an expression such as $A\sin\omega t + B\cos\omega t$ can be written as $R\sin(\omega t \pm \phi)$. Some of the mathematics involved is covered on Theory Sheet 2.

The Tutorial

It is advisable, but not essential, that you should have already completed Tutorial Sheet 1.

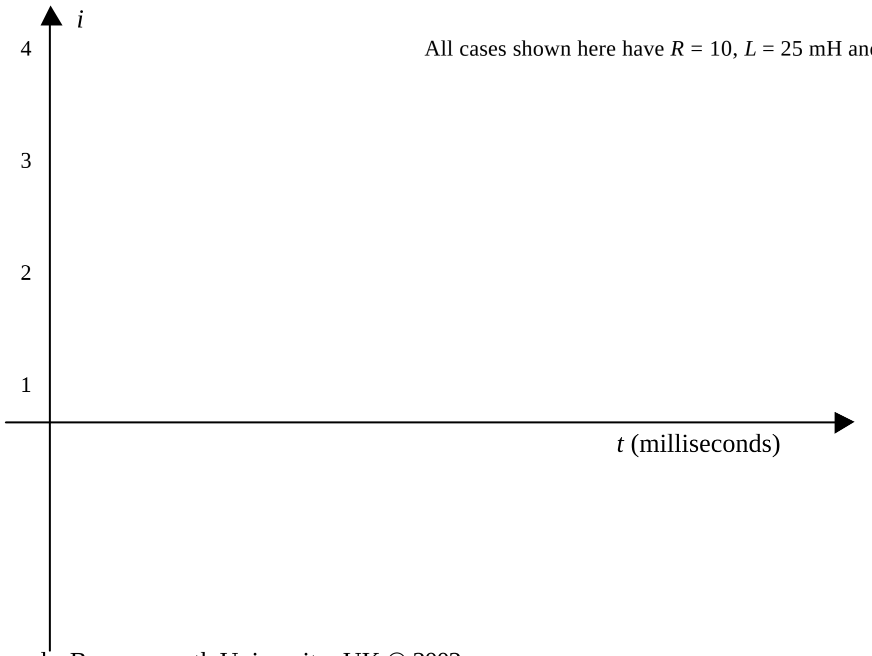
(a) Changing *a*

Ensure the default values i_o = 4, L = 25 and R = 10 are set and choose "Sinusoidal" ($e(t) = a\sin\omega t$) from the drop-down box on the right of the window. The default sinusoidal function is $e(t)$ = 20sin400t. If you have already used the "Trigonometric Functions" applet from ***MathinSite***, you will know that a is the amplitude of this applied alternating voltage (20 volt) and ω is its angular velocity (400 rads/sec - giving a frequency of 400/2π Hz and a periodic time of 2π /400 seconds).

For the sake of clarity later in this exercise leave a as 20, but change ω to 500.

On the axes below, draw a scale on the horizontal axis, sketch the graph that you see on-screen, write "$e(t)$ = 20sin500t" alongside this curve and, in the table on the following page, note how the information for this solution curve has been entered.

- Ensure "Multiple Plots" is selected from the "Graph" menu and change the value of a by clicking *on the arrow at the relevant end of* the a slider bar.
- Repeat so that your plot shows the graphs for a = 0, 10, 20, 30 and a = 40.
- Superimpose all the curves on the axes and label each "$e(t)$ = $a$sin500t" with the relevant value of a.



If you have completed Tutorial Sheet 2 you will not be surprised to note that sinusoidal input results in sinusoidal steady state output.

Remember that *steady state* does not mean *constant* *value*. It means that the output settles down to a constant *state* with a mathematical solution that holds for all time – theoretically. A steady sinusoidal output in this case.

With applied input voltage, $e(t) = a \sin 500t$ ……

a (V)	R (Ω)	L (H)	R/L	Current, i, at t seconds (A)	t (s)	$5\times t$ (ms)
0	10	25×10^{-3}				
10	10	25×10^{-3}				
20	10	25×10^{-3}	400	$i = 4.976e^{-400t} - 0.976\cos500t + 0.78\sin500t$	$\frac{1}{400}$	12.5
30	10	25×10^{-3}				
40	10	25×10^{-3}				

Remember that the solution of the governing differential equation comes in two parts. For the example shown in the table, these two parts are $4.976e^{-400t}$ and $-0.976\cos500t + 0.78\sin500t$.

- The exponential part of the solution relates directly to the system components L and R and is called the ***Complementary Function*** (CF). The CF gives the ***transient*** *response*.
- The sinusoid part of the solution relates directly to the type of input and is called the ***Particular Integral*** (PI). The PI gives the ***steady state*** *response*.

With respect to the applied voltage, what did the case $a = 0$ represent?

……………………………………………………………………………………………………

Write down here the CF part of the solution for each of the above five cases.

……………., ……………., ……………., ……………., …………….,

State here the value that remained the same for all five CFs.

……………………………………………………………………………………………………

How did the value of $5\times t$ change for the different values of a?

……………………………………………………………………………………………………

How are the two previous questions related in terms of the transient decay time?

....

Write down here the particular integral (PI) part of the solution for the case $a = 20$ (remember this part relates only to the circuit's 'steady state' response).

....

What is the equivalent value of angular velocity (w) for this part of the response?

....

Bearing in mind that the periodic time (and hence frequency) of oscillations is related to the value of w, what is the relationship between the input angular velocity (hence periodic time and frequency) and the circuit's response angular velocity?

....

Is the response angular velocity affected by changing the value of a? Yes / No.........

If it is, how? If it isn't, what is? …………...

The points where the steady state oscillations cross the time axis are the same in all five cases investigated. Based on what you have found above, why is this?

....

(b) Changing w

Reset the default values $i_o = 4$, $L = 25$ and $R = 10$ and $e(t) = 20\sin 400t$.

From the above you will know that changing a, the amplitude of the applied voltage changes the amplitude of the steady state sinusoidal current only (and, perhaps, the phase angle – see the Exercises below). Varying the input angular velocity w, then, should only affect the ……………………………

There are no boxes to fill in here. Just move the w slider whichever way you want and notice how the steady state varies. 'Single Plot' mode should be fine, but you may want to use 'Multiple Plot' to see this better.

Describe here the effect that varying w has on the plots.

....

Follow the amplitudes of the resulting steady state oscillations.
Did the amplitude change? Yes / No ....
Is this what you expected? Yes / No .... (now see the Exercises below)

Summary

With an applied sinusoidal voltage, the LR circuit equation is $L\frac{di}{dt} + Ri = a\sin wt$

- **The solution (current) for the LR circuit with an applied sinusoidal voltage is always of the form:**

$$i(t) = Ae^{-kt} + B\sin wt + C\cos wt$$

It contains two parts:

  - **The Complementary Function (CF), Ae^{-kt}, is the *transient* response. $-kt$ depends on the circuit parameters L and R only, so dictating that the time for transient decay is dependent only on the circuit itself.**
  - **The Particular Integral (PI), $B\sin wt + C\cos wt$, a sinusoidal output, is the *steady state* response.**
- **Sinusoidal input implies sinusoidal steady state (long-term) output.**
- **Since w is the same for both output and input, the frequency and periodic time of the resulting steady state current are the same as those of the applied voltage.**
- **Pushing a sinusoidal applied voltage through an LR Series Circuit can introduce a phase difference between the input and the steady state output. (Theory Sheet 2)**
- **If either of a or w is zero, the applied voltage is zero and so the steady state output is zero. (*Zero input implies zero steady state output*)**

Exercises

1. When using the applet with the above tutorial, you may have spotted that changing the input, w, of the applied sinusoidal voltage also changed the steady state *amplitude*. You perhaps didn't expect this! What you might not have expected also is that changing L and R also affects the amplitude of the steady state oscillatory current!

Your exercise here is to verify these comments by varying the values of w, L, R and i_0 in the applet and identify which, if any, of these inputs affects each of the amplitude, the angular velocity and the phase angle of the steady state oscillations.

2. Solve the complete <u>*general*</u> differential equation

$$L\frac{di}{dt} + Ri = a\sin wt \qquad \text{given } i = i_0 \text{ when } t = 0,$$

to see how the steady state amplitude is also affected by changing the system parameters and the value of w (as was done in order to produce the mathematics that drives the applet).

Does your solution indicate whether the steady state oscillations are affected by changing the initial condition, $i(0) = i_0$?

Identify from your general solution, which, if any, of the inputs affects the amplitude, the angular velocity and the phase angle of the steady state oscillations.