Name ……………………………………… Course ……………………………….
Print and use this sheet in conjunction with ***MathinSite***'s 'LR Series Circuit' applet and theory sheet.

The LR Series Circuit
Tutorial Sheet 4
-with applied exponential voltages, $e(t) = ae^{pt}$

Learning Outcomes, **Introduction**, **Loading the Software** and **Running the applet** have all been covered in the previous LR Series Circuit sheets.

The Tutorial

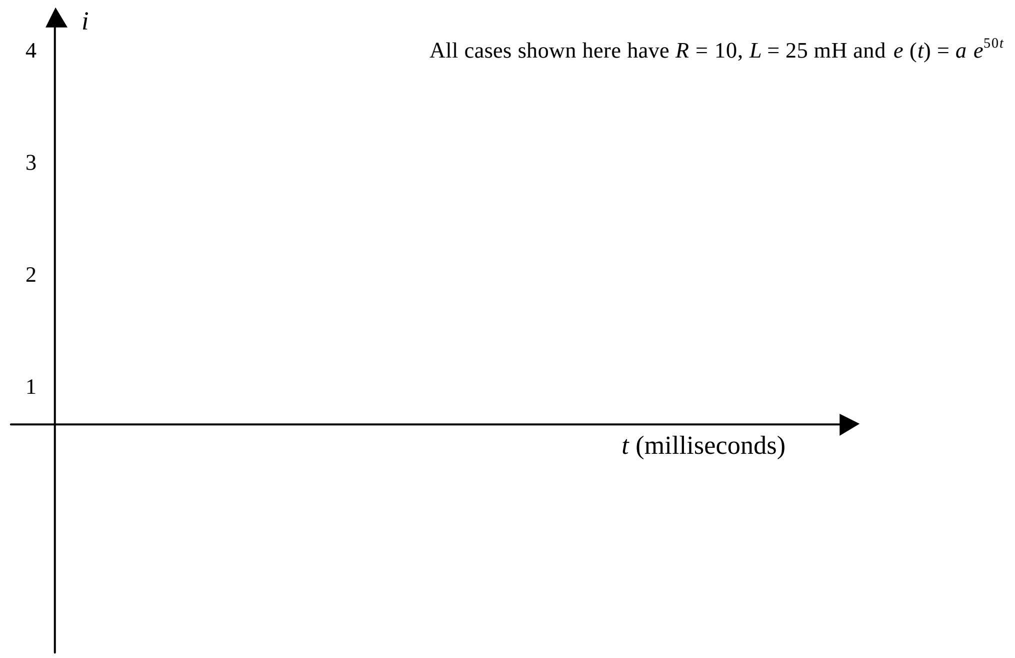
It is advisable, but not essential, that you should have already completed at least Tutorial Sheet 1.

(a) Changing *a*

Ensure the default values $i_o = 4$, $L = 25$ and $R = 10$ are set and choose "Exponential" ($e(t) = a.\exp(pt)$) from the drop-down box on the right of the applet window. The default exponential applied voltage is $e(t) = 10e^{50t}$.

On the axes below, draw a scale on the horizontal axis, sketch the graph that you see on-screen, write $e(t) = 10e^{50t}$ alongside this curve and, in the table on the following page, note how the information for this solution curve has been entered.

- Ensure "Multiple Plots" is selected from the "Graph" menu and change the value of a by clicking *between the slider and the arrow at the relevant end of* the a slider bar.
- Repeat, so that your plot shows all the graphs for a = -5, 0, 5, 10, 15 and a = 20.
- Superimpose all these curves on the graph below and label each $e(t) = ae^{50t}$ using the relevant value of a.



If you have completed Tutorial Sheets 2 and/or 3 you will not be surprised to note that exponential input results in exponential steady state output.

Remember that *steady state* does not necessarily mean steady, or *constant*, *__value__*. It means that the output settles down to a constant *__state__* with a mathematical solution that holds for all time – theoretically. A steady exponential output in this case.

With applied input voltage, $e\ (t) = a\ \mathrm{e}^{50t}$ …

a (V)	R (Ω)	L (mH)	R/L	Current, i, at t seconds (A)	t (s)	$5\times t$ (ms)
-5						
0						
10	10	25	400	$i = 3.111e^{-400t} + 0.889e^{50t}$	$\frac{1}{400}$	12.5
15						
	10	25				

Remember that the solution of the governing differential equation comes in two parts. For the example shown in the table, these two parts are $3.111e^{-400t}$ and $0.889e^{50t}$.

- The first exponential (decay) relates directly to the system components L and R and is called the ***Complementary Function*** (CF). The CF gives the ***transient*** *response*.
- The second exponential relates directly to the type of input and is called the ***Particular Integral*** (PI). The PI gives the ***steady state*** (*long-term*) *response*.

With respect to the applied voltage, what did the case $a = 0$ represent?

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In physical terms, what does the case $a = -5$ represent?

………………………………………………………………………………………………

Write down here the CF part of the solution for each of the above six cases.

………….., ………….., ………….., ………….., ………….., …………..

State here the value that remained the same for all six CFs.

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How did the value of $5\times t$ change for the different values of a?

………………………………………………………………………………………………

How are the two previous questions related in terms of the transient decay time?

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Write down here the particular integral (PI) part of the solution for the cases when $a = 5$ and $a = -5$ (remember this part relates only to the circuit's 'steady state' response).

...., ...

Use these results to explain any similarities/differences between responses for each case.

....

....

What is the relationship between the coefficients (numbers multiplying) of the two exponentials and the initial condition?

....

Why is this? (Hint: think $t = 0$)

....

(b) Changing *p*

Reset the default values $i_o = 4$, $L = 25$ and $R = 10$ and $e(t) = 10e^{50t}$.

For an applied voltage ae^{pt}, varying p should only affect the CF / PI

(delete as necessary)

There are no boxes to fill in here. Just move the p slider whichever way you want and notice how the steady state varies. 'Single Plot' mode should be fine, but you may want to use 'Multiple Plot' to see this better.

Describe here the general effect on the plots (responses) of varying p.

....

....

....

In particular, note below the differences between the cases:

$p > 0$ ....

$p = 0$...

$p < 0$...

Varying p, which *should* only affect the PI, also has some effect on the CF (transient). Note here, using the solution given in the text area of the applet, which part of the CF (transient) is affected, and which isn't.

Affected ....

Not affected...

Summary

With an applied exponential voltage, the LR circuit equation is $L\frac{di}{dt} + Ri = ae^{pt}$

- **The solution (current) for the LR circuit with an applied exponential voltage is of the form:**
  - ❑ $i(t) = Ae^{kt} + Be^{pt}$ **when** $k \neq p$ **(**$k < 0$ **always)**

    OR
  - ❑ $i(t) = (A + Bt)e^{kt}$ **when** $k = p$ **(**$k < 0$ **always)**

  It contains two parts:
  - **The Complementary Function (CF),** Ae^{kt}**, is the *transient* response.** k **(< 0) depends on the circuit parameters** L **and** R **only, so dictating that the time for transient decay is dependent only on the circuit itself.**
  - **The Particular Integral (PI),** Be^{pt} $(k \neq p)$ **or** Bte^{pt} $(k = p)$**, an exponential output, is the *steady state* response. In this particular case, the steady state response is necessarily zero. (Why?)**
- **Exponential input implies exponential steady state (long-term) output.**
- **If** $a = 0$**, the applied voltage is zero and so the steady state output is zero. (*Zero input implies zero steady state output*)**
- a **'amplifies' vertically the PI's exponential response** e^{pt} **for** $a > 1$ **or** $a < -1$ **and compresses it vertically when** $-1 < a < 1$**.**
- $a < 0$ **produces a steady state response mirrored across the time axis compared with the equivalent case when** $a > 0$**.**
- **If** $p = 0$**, the applied voltage is** $ae^{0t} = a$ **and so the steady state output is a constant value. (*Constant input implies constant steady state output*)**
- **If** $p < 0$**, the steady state response is necessarily zero. (Why?)**
- **If** $p > 0$**, the steady state is unbounded exponential growth.**

Exercises

1. Set up the circuit with i_0 = 2 A, L = 25 mH and $R = 1\Omega$. Set up the exponential input with a = 10 V. Now move the p slider until the $i(t) = (A + Bt)e^{kt}$ type of solution is obtained. Move the p slider either side of this value and sketch the graphical response at and around this p value. Label each graph with its solution. Did the special case have a response that did not fit in with the sequence of curves surrounding this case? What was the steady state solution in each case?

2. When using the applet with the above tutorial, you may have spotted that changing L and R also affects the steady state exponential current!

Your exercise here is to vary the values of a, p, L, R and i_0 in the applet and identify which, if any, of these inputs affect the different parts (transient (CF) and steady state (PI)) of the output.

3. Solve the complete *general* differential equation

$$L\frac{di}{dt} + Ri = ae^{pt} \qquad \text{given } i = i_0 \text{ when } t = 0, \qquad \text{for the case when } p \neq -\frac{R}{L}$$

to see how each part of the output is affected by changing the system parameters, the applied voltage and the initial condition (as was done to produce the mathematics that drives the applet).

From your analytical result here, you should be able to corroborate the results you found in Q. 2.

4. Set up the circuit with i_0 = 5 A, L = 40 mH and $R = 1\Omega$. Set up the exponential input with a = 3 V and p = 50. You may have to maximise the window and/or move the graph vertically to see the response. Sketch the graph. Identify on your graph which part of the mathematical solution affects which part of the graph (although note that the transient 'officially' lasts a full 200 ms approximately).

Now move the i_0 slider through 4.5, 4.0, etc, noting how the mathematical solution changes in each case, stopping at i_0 = 1.0. Note the solution in this particular case - it is different from the others. Comment and explain. Is the last line in the text area of the applet still relevant in this case?

Use your answer to Q.3 to identify why, for L = 40, R = 1, a = 3 and p = 50, i_0 = 1.0 is a special case.