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Use these sheets whilst using *MathinSite*'s Parabola applet for a permanent record of your work.

$y = ax^2 + bx + c$, the Equation of the Parabola
The Software Tutorial

Learning Aims

- To develop, through experimentation, an understanding of the equation of the parabola and a feel for the effect of changing its parameters.
- To answer "what if ...?"-type questions about the parabola and the parabolic equation.

Introduction

The equation $y = ax^2 + bx + c$ is the equation of a *parabola*.

x and y are *variables*, since across the graph of the parabola, as x varies, y varies.

x is called the *independent variable* (the horizontal axis on the graph) and y is called the *dependent variable* (the vertical axis). The values of y *depend on* the value of x.

a, b and c are *parameters*. Although it is also possible to choose values for a, b and c arbitrarily, for any one particular parabola their values will not change. A new set of values for a, b and/or c will result in a *different* parabola.

The graph of $y = ax^2 + bx + c$ takes the shape of the cross-section of a bowl, opening either upwards or downwards. The lowest point when it opens upwards (or the highest point when it opens downwards) is called the *vertex*. All parabolas with the above equation will cross the y-axis *somewhere*, but need not necessarily cross the x-axis

Here y is a function of (i.e. depends on) x, written $y = f(x)$ in general, so here we can write $f(x) = ax^2 + bx + c$. In this case, $f(x)$ is called the *quadratic function*.

The quadratic function has important applications in the mathematical analysis of topics in, amongst others, science, technology and business studies – in particular in cases where there is a *square-law relationship* between two sets of variables. For example,
- the cross-sectional shape of a car's headlamp reflector is parabolic, and
- the aerodynamic drag of a moving vehicle is proportional to its velocity *squared*.

The software allows investigation of the parabolic function, $y = ax^2 + bx + c$

Loading the Parabola Applet

The parabola applet is obtained from the web site from which this work sheet was downloaded. When the applet is loaded, you will see something similar to the display shown here.
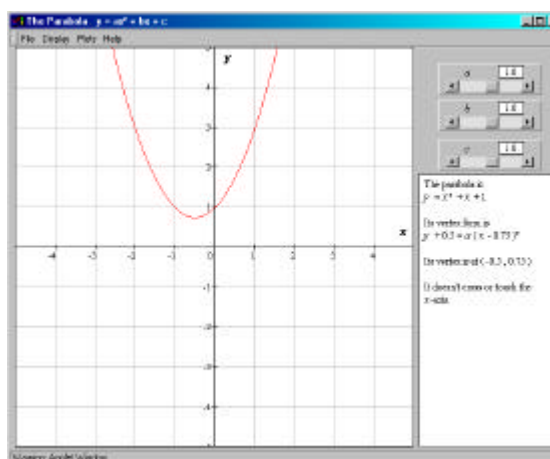
Running the applet

The software uses the default values $a = 1$, $b = 1$ and $c = 1$. With these default values, the relevant graph (in

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this case $y = x^2 + x + 1$) is displayed in the main graphics area.

The three slider bars on the right of the applet window allow the user to change each of the parameters of the parabola (here a, b or c). Clicking the arrow boxes at either end of the slider bar effect *small changes* in the parameter values. Clicking in the main part of the box of the slider bar makes *larger changes. Clicking, holding and moving the <u>slider</u> in the slider bar can make **any** sort of change*.

Changing the values by moving the sliders is the only means of user-input.

The software gives you the option of showing single plots or multiple plots (under 'Plots' in the pull down menu structure at the top of the applet window). It also allows you to 'drag' the plot around (to see areas not covered by the original window on the plot) by holding down the mouse button whilst at the same time moving the mouse over the plot. The plot can be recentred at any time using the 'Centre on Origin' option, under 'Display'. You can reset the software at any time to the default values (those used when the program loads) using 'Reset' from the 'Plots' menu.

<u>The Tutorial</u>

<u>The effect of changing *a*</u>
- With the default values $a = 1$, $b = 1$ and $c = 0$, sketch the graph that you see on-screen on the axes below and write alongside this line its equation.
- Choose "Multiple Plots" from the "Plots" pull-down menu and change the value of *a to* $a = 2$ by clicking once *inside* the *a* slider bar to the right of the slider itself.
- Repeat so that your plot shows also the graphs for $a = 3$, $a = 4$, $a = 5$ and $a = 6$.
- Superimpose all the lines on the axes below, labelling each line with its equation.
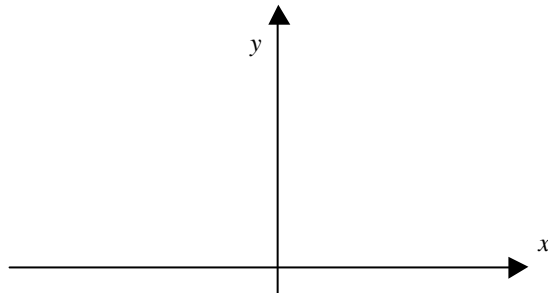
Where did each parabola cross the *y*-axis?

....

Where was the vertex of the parabola in each case?

....

What was the effect on the line of increasing *a* from 1 to 6?

....

Where did these parabolas cross the *x*-axis?

....

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- Reset the graphic display to the default setting by choosing "Reset" from the "Plots" pull-down menu.
- With the default values $a = 1$, $b = 1$ and $c = 1$, sketch again the graph that you see on-screen on the following axes and label it. (Don't forget, label the axes too)
- If not already chosen, choose "Multiple Plots" from the "Plots" pull-down men and change the value of *a to a* = 0.9 by clicking once *on the arrow* at the left-hand end of the *m* slider bar.
- Repeat this so that your plot shows also the graphs for $a = 0.8$, $a = 0.7$, $a = 0.6$ and $a = 0.5$.
- Superimpose each parabola on the axes below, labelling each with its equation.
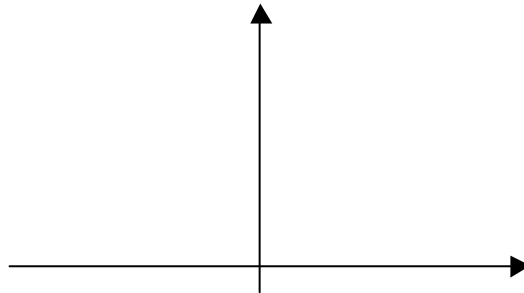
Where did each parabola cross the *y*-axis?

....

Where was the vertex of the parabola in each case?

....

What was the effect on the parabola of decreasing *a* from 1 to 0.5?

....

So on all plots so far, what was changing as you changed *a*?

....

... and what didn't change?

....

In the light of what you have found so far, what do you guess would be the effect of making the value of *a* equal to zero?

..   Try it now.

- Now click on the slider of the *a* slider bar, hold down the mouse button and move the mouse to the left until the value of *a* is -1

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- You may need to clear all other plots on-screen before proceeding. If so, this is done using "Clear Multiple Plots" from the "Plots" pull-down menu.
- Sketch again the graph that you see on-screen on the following axes and label it (and label the axes too).
- If not already chosen, choose "Multiple Plots" from the "Plots" pull-down menu and change the value of *a* to *a* = -2 by clicking once *inside* the *a* slider bar to the left of the slider itself.
- Repeat this for *a* = -3, *a* = -4, *a* = -5 and *a* = -6.
- Superimpose each parabola on the axes below, labelling each with its equation.
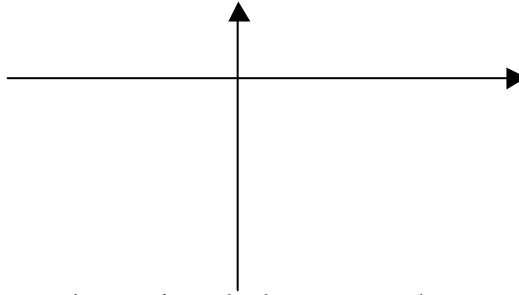
Where did each of these cross the *y*-axis and where was each vertex?

....

What was the effect on the parabola of decreasing *a* from -1 to -6?

....

What is the overall effect of making the value of *a* negative?

....

Superimpose on the previous axes the graphs for which *a* = -0.9, *a* = -0.8, *a* = -0.7, *a* = -0.6 and *a* = -0.5 (all with *b* = 1 and *c* = 1). Use the applet if you can't guess.

Why do you think that the applet stops at *a* = 10 in the positive direction and *a* = -10 in the negative direction?

....

The effect of changing *c*

- Reset the graphic display to the default setting by choosing "Reset" from the "Plots" pull-down menu.
- With the default values *a* = 1, *b* = 1 and *c* = 1, sketch again the graph that you see on-screen on the following axes and label it. (Don't forget, label the axes too)
- If not already chosen, choose "Multiple Plots" from the "Plots" pull-down men and change the value of *c to c* = 2 by clicking once *inside the slider* to the right of the slider in the *c* slider bar.
- Repeat this so your plot shows also the graphs for *c* = 3, *c* = 4, ... as far as you need in order to understand the effect of increasing *c*
- Sketch the on-screen family of curves on the following diagram.

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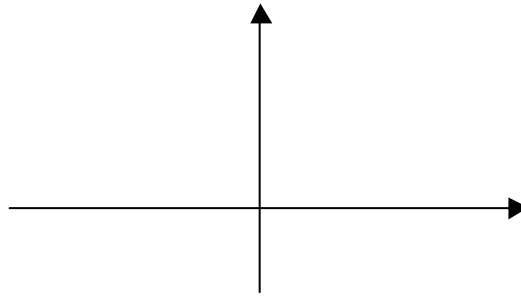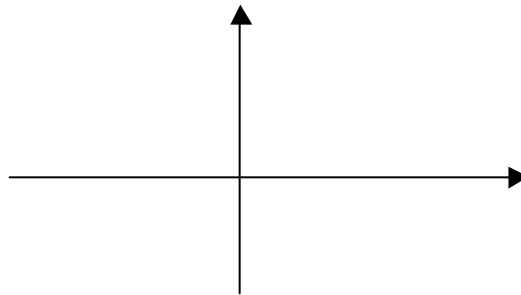

(a) Describe the effect of increasing the value of *c*. (b) What happened to the overall shape of the parabolas during this time? (c) What can you say about the value of the *y*-intercept in each case

(a)..

(b)..

(c)..

Now consider *c* with negative values. Reset and try *c* = -1, *c* = -2, … Sketch and label the curves along with their *y*-intercepts.

Did you notice that you haven't tried *c* = 0? Before using the applet, write below what you think is special about any parabola with *c* = 0.

....

Describe in the space below the parabola obtained when both *a* = 0 and *c* = 0? (It may help to write down the equation of the 'parabola' in this case.) Now use the applet plot it and see.

....

The effect of changing *b*

- Reset the graphic display to the default setting by choosing "Reset" from the "Plots" pull-down menu.
- With the default values *a* = 1, *b* = 1 and *c* = 1, sketch again the graph that you see on-screen on the following axes and label it. (Don't forget, label the axes too)
- If not already chosen, choose "Multiple Plots" from the "Plots" pull-down men and change the value of *b* to *b* = 2 by clicking once *inside the slider* to the right of the slider in the *b* slider bar.

For the associated 'Parabola' applet, go to http://mathinsite.bmth.ac.uk/html/applets.html

- Repeat this so your plot shows also the graphs for $b = 3$, $b = 4$, ... as far as you need in order to understand the effect of increasing b.
- Sketch the on-screen family of curves on the following diagram.
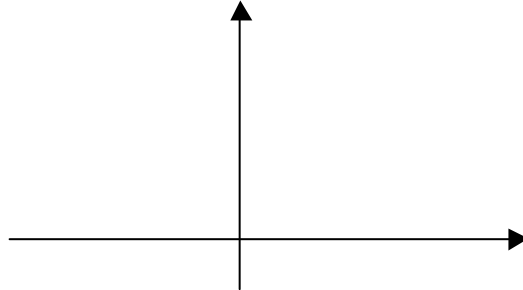
(a) What happened to the overall shape of the parabolas during these changes? (b) What can you say about the value of the y-intercept in each case (c) Describe the effect of increasing the value of b.

(a)..
(b)..
(c)..

Now consider c with negative values. Reset and try $b = -1$, $b = -2$, ... Sketch and label the curves along with their y-intercepts.
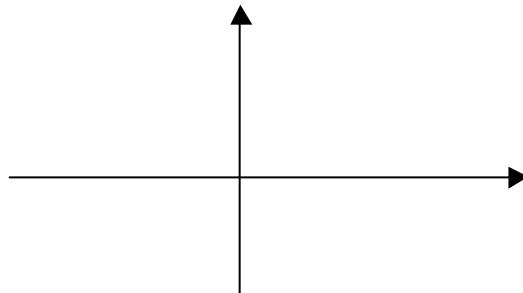
Did you notice that you haven't tried $b = 0$? Before using the applet, write below what you think is special about any parabola with $b = 0$.

....

So changing the value of *a* alone changes only the shape of the parabola,

changing the value of *b* alone changes only the position of the vertex

and changing the value of *c* alone only moves the parabola up or down.

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Exercises

1. *Without* using the software, sketch$^{\oplus}$ the following parabolas:

   $$y = x^2 \qquad\qquad\qquad y = x^2 + 1$$
   $$y = 2x^2 \qquad\qquad\qquad y = 2x^2 - 4$$

   Verify your sketches using the software.

2. The co-ordinates of the point of intersection of any parabola with the x-axis can be found by letting $y = 0$ in the parabola's equation, i.e. $ax^2 + bx + c = 0$. This quadratic equation can be solved by factorisation, or by using the well-known formula

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   For example, $y = x^2 + 3x + 2$ will cross the x-axis when $x^2 + 3x + 2 = 0$. This factorises as $(x + 1)(x + 2) = 0$, so $x + 1 = 0$ or $x + 2 = 0$ giving $x = -1$ or $x = -2$. So the given parabola crosses the x-axis at points with co-ordinates (-1, 0) and (-2, 0). Use the applet to verify this.

   Alternatively, using the formula with $a = 1$, $b = 3$ and $c = 2$,

   $$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 2}}{2 \times 1} = \frac{-3 \pm \sqrt{1}}{2} = \frac{-3 \pm 1}{2} \text{ giving } x = -1 \text{ or } x = -2 \text{ again.}$$

   Do the same for
   (a) $y = x^2 - 5x + 6$
   (b) $y = 4x^2 - 4x + 1$ and
   (c) $y = x^2 + 4x + 13$

   Note in particular what happens in (c).
   Use the accompanying applet to verify your answers.

3. In certain, simpler cases, it not necessary to use either factorisation or the formula method to solve the equation $y = 0$. Determine the co-ordinates of the point at which the parabolas in Q.1 cross the x-axis. Use the applet to verify your solutions.

$^{\oplus}$ Here 'sketch' means roughly draw the axes with the straight lines superimposed. You should label the axes and each graph and indicate any important points (such as the y-intercept). 'Plot', on the other hand, means 'plot points accurately on a properly scaled and labelled graph' using data from a table of values. You should label the graphs and important points in either case.

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