

## Projectiles 1:

### Projectiles in a non-resisting medium.

### The Software Tutorial.



#### Learning Outcomes

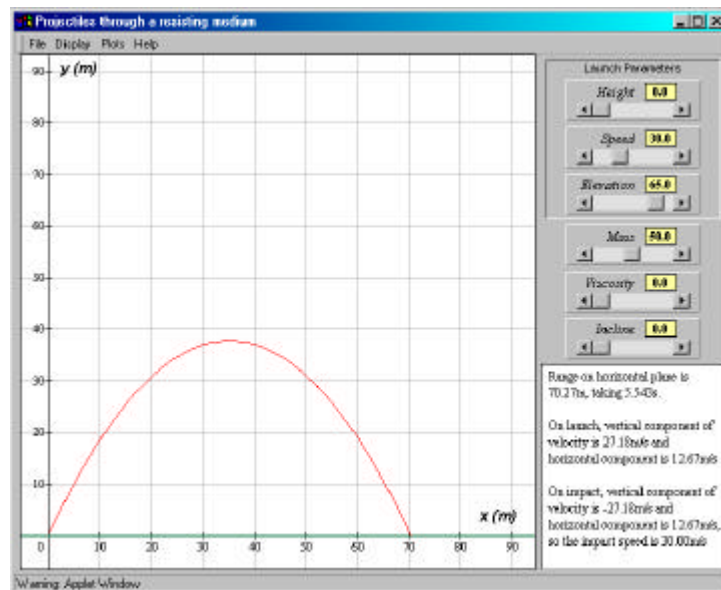
- To develop, through experimentation, a deeper understanding of the motion of a projectile through a non-viscous medium and a feel for the effect of changing the associated projectile parameters.
- To enable questions about such projectiles of a "what if ...?" nature to be answered.

#### Introduction

'Projectiles' is a subject studied in sixth form and undergraduate level Mathematics, Science or Technology. Usually a simplified model is studied where a point object is projected through a non-viscous medium, so that the classical parabolic trajectory results. No allowance is made for aerodynamic drag and so the more realistic trajectory that is associated with projectiles 'in the real world' is not seen. This software tutorial should help deepen your understanding of how the trajectory of the projectile varies as parameters are changed; namely, mass, launch height, velocity and angle of elevation. It is also possible to investigate the trajectory over an inclined plane.

#### Loading the Software

Load the 'Projectiles' applet from the web site from which this document was taken. When loaded, you will see something similar to the applet window shown here.



#### Running the applet

The software uses the default values

- *Height* = 0 metres, (height from which projectile is launched)
- *Speed* = 30 m/s, (initial speed and ...)
- *Elevation* = 65°, (... angle to the horizontal at which body is projected)
- *Mass* = 50 kg, (mass of the projectile)
- *Viscosity* = 0, (of the medium through which projectile passes – for a non-resisting medium, the viscosity is zero)
- *Incline* = 0, (this allows you to project the body across an inclined plane)

The six slider bars on the right of the applet window allow the user to change each of the above parameters. Clicking the arrow boxes at either end of the slider bar effect *small*

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*changes* in the parameter values. Clicking in the main part of the box of the slider bar makes *larger changes*. Clicking, holding and moving the slider in the slider bar can make *any sort of change*.

***Changing values by moving the sliders is the only means of user-input.***

The software gives you the option of showing single plots or multiple plots (under 'Plots' in the pull down menu structure at the top of the applet window). It also allows you to 'drag' the plot around (to see areas not covered by the original window on the plot) by holding down the mouse button whilst at the same time moving the mouse over the plot. The plot can be re-centred at any time using the 'Centre on Origin' option, under 'Display'. You can reset the software at any time to the default values (those used when the program loads – given above) using 'Reset' from the 'Plots' menu.

## **The Tutorial**

**Changing Mass** Use the default values provided when the applet loads (use “Reset” from the “Plots” pull-down menu if you have already changed any of the sliders!). Now move the *Mass* slider to the right, by clicking on the arrow at the right-hand end of the *Mass* slider bar and holding down the mouse button. You can move it to the left in a similar manner. Try this with the velocity vectors ‘switched on’ for every second of the trajectory (from the “Display” – “Show Velocity” – “Every Second” menus).

What effect did changing the mass have on the trajectory and the velocities every second?

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***So for the non-viscous case, the size of the mass is irrelevant***

This result is similar to that found by Galileo in his famous experiment involving dropping weights from the top of the Leaning Tower of Pisa, namely: *different weights fall (or travel the same trajectory) at the same rate.*

Galileo's famous experiment can be simulated here. Switch off the velocity vectors (from the “Display” – “Show Velocity” – “Off” menu) and choose “Reset”, “Single Plot”, “Centre on Origin”. Put *Speed* = 0 and the angle of elevation to -90 (i.e. the object is *dropped* vertically downwards – although with *Speed* = 0 this is not absolutely necessary. Think about it!). Choose an initial height of 60. Now choose cases for which only *Mass* changes, in particular, try values of *Mass* = 10, 20, 30, 40 and 50 by clicking 5 times in the *Mass* slider bar to the right of the slider itself. Make a note for each case in the spaces below, the times that the object takes to fall the 60 metres.

10..... 20..... 30..... 40..... 50.....

However, as you will see in the second Projectiles tutorial, Galileo's conclusion was incorrect (or, more to the point, he couldn't detect the small differences in time with the masses and distances he was using).

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In this tutorial, the *Viscosity* slider will always be set to zero – this is the non-viscous case and, as such, is not very realistic in terms of what happens in the “real world” since even air has some viscosity. So why investigate this case when it is not truly representative? Because the mathematics falls well within the A-level or first-year undergraduate syllabus. In the second Projectiles tutorial sheet (*Projectiles 2: Projectiles through a resisting medium*) you investigate a more realistic case.

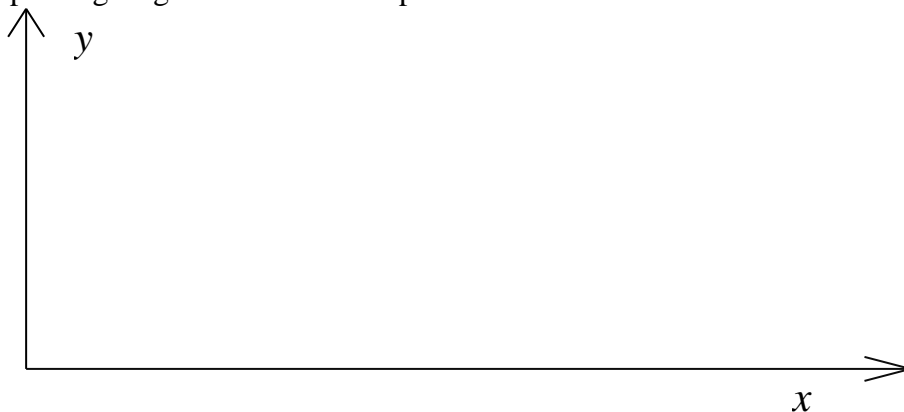
**Changing initial height** Use the slider bars to set up  $Height = 0$ ,  $Speed = 30$  and  $Elevation = 60$  and sketch the resulting trajectory on the axes below. Use “Multiple Plots” from the “Plots” menu and change  $Height$  to 10, 20 and 30. Sketch the trajectories below and indicate the values of the initial height and the corresponding range on the horizontal plane.



Describe the effect on the range of projecting the body from higher positions.

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**Changing initial velocity** Use the slider bars to set up  $Height = 0$ ,  $Speed = 10$  and  $Elevation = 60$  (remember in this tutorial sheet, *Viscosity* is always set at zero and so the value of *Mass* is irrelevant). Use “Clear Multiple Plots”, “Centre on Origin” as necessary from the pull-down menus and superimpose the curves for which  $Speed = 20, 30$  and  $40$ . (you may have to move the graph to the right if you want to see where the projectile lands for one of these cases – click and hold the cursor on the graphics area and pull the graph to the left). Sketch the trajectories below and indicate the values of the initial velocities and the corresponding range on the horizontal plane.



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Describe the effect on the range of projecting the body at different speeds.

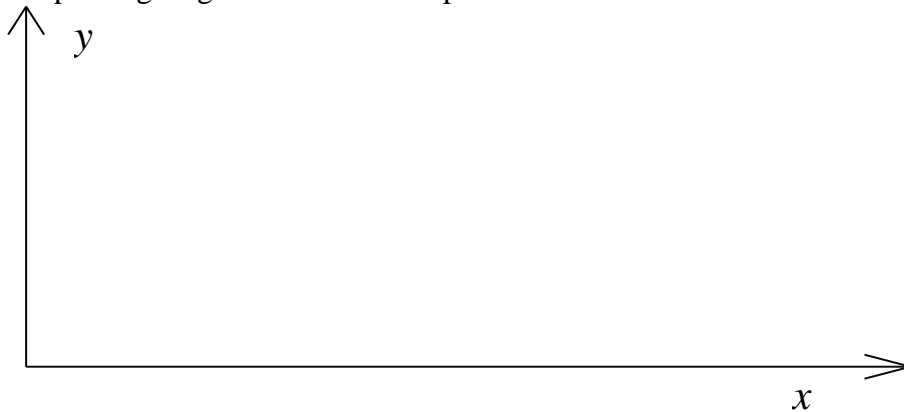
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Is this always going to be the case? If so, why? If not, why not?

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(To help you answer the previous question, you might like to try increasing *Speed* - 40, 50, etc to 100 - and note how the range varies for these cases.)

**Changing angle of projection** Use the slider bars to set up *Height* = 0, *Speed* = 30 and *Elevation* = 20 (remember in this tutorial sheet, *Viscosity* is always set at zero and the value of *Mass* is irrelevant). Use "Clear Multiple Plots", "Centre on Origin", "Multiple Plots" as necessary from the pull-down menus and superimpose the curves for which *Elevation* = 30 and 40. Sketch the trajectories below and indicate the values of the initial velocities and the corresponding range on the horizontal plane.



Describe the effect on the range of projecting the body at angles in the range 20° to 40°.

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Is this always going to be the case? If so, why? If not, why not?

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(To help you answer the previous question, you might like to try increasing *Elevation* to 40, 50, 60 and 70 - and note how the range varies for these cases.)

Hmm. What's happened to the range now?

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Use the arrows at either end of the Elevation slider bar to determine the angle in this case that provides the maximum horizontal range.

For an initial speed of 30 m/s,

Max Range of ..... metres occurs when elevation is ..... degrees.

Will this angle of elevation always give maximum range? Should you really extrapolate from the results of one test? Repeat the previous experiment but with *Speed* =50. This time the trajectory disappears off to the right of the graphics area. Don't bother chasing where the projectile reaches ground level on the graphics; just check the 'Range', first line in the text-box on the right, to determine when you obtain the maximum range.

For an initial speed of 50 m/s,

Max Range of ..... metres occurs when elevation is ..... degrees.

### The Velocity Vectors

Choose from the menu system, "Reset", "Centre on Origin", "Single Plots" as necessary. Under the "Display" menu, choose "Show Velocity" – "Every .5 sec". The graphics displays the horizontal and vertical components of the velocity of the projectile at every half second through its trajectory.

What do you notice about the horizontal component of velocity throughout the motion?

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Why do you think this is so?

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What do you notice about the direction of the vertical components of velocity either side of the maximum height?

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(Easy one – no trick!) Why do you think this is so?

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What will be the vertical component of the velocity at maximum height? (You can *almost* see this if you keep the 0.5 second gap and change *Speed* to 41 and *Elevation* to 73)

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Under the "Display menu", choose "Show Velocity" – "Off" and then "Show Max Height".

For the case that you have showing on the graphics, write down:

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initial speed = ..... m/s

impact speed = ..... m/s

horiz distance to max ht = ..... m

Range = ..... m

time to max. ht = ..... seconds

time of flight = ..... seconds

The time from when the projectile attains maximum height to the point where it hits the ground is, therefore, ..... seconds.

From this information, what can you say about the trajectory with respect to a vertical line drawn through the point of maximum height?

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The trajectory is in fact a parabola with its vertex at the position of maximum height.

This doesn't happen in the real world. What else have we ignored, other than viscosity, that will affect the trajectory of a projectile in real life? (List at least two.)

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Give an example of the trajectory of a free-flying object that does not behave this way (think sport!) and give a brief explanation (look at the previous question) of why it doesn't exhibit the parabolic trajectory indicated here by this software (with *Viscosity* = 0).

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### **Changing the Angle of the Inclined Plane**

Certain mathematics syllabuses include investigation of the trajectory of a projectile over an inclined plane. While this adds little to the graphical investigation of projectiles (but plenty to the mathematical analysis), the software here can amplify certain - albeit common sense - considerations. The software can also be used to check answers to inclined plane projectile problems.

Choose "Reset" and "Show Max Height", then "Centre on Origin" and "Single Plot" if necessary.

Increase the value of *Incline*, 10° at a time, from 10° to 40°. Note the value of the maximum height obtained. Now increase *Incline* to 50°. What happened to Max Height and why?

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Now increase *Incline* to 60° and then finally notice what happens as you change the angle of incline to 70°. What happened and why? (The clue is in the text box on the right of the screen!)

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