

Projectiles 2: Projectiles in a Resisting Medium. The Software Tutorial.



Learning Outcomes

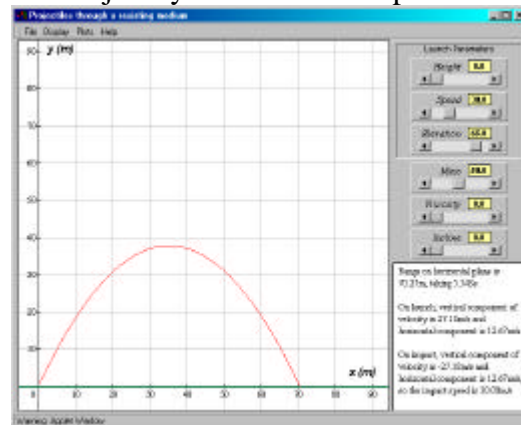
- To develop, through experimentation, a deeper understanding of the motion of a body projected through a resisting medium and a feel for the effect of changing the associated parameters.
- To enable “what if ...?” questions about such projectiles to be answered.

Introduction

‘Projectiles’ is a subject studied in sixth form and undergraduate level Mathematics, Science or Technology. Usually a simplified model is studied where a point object is projected through a non-viscous medium, so that the classical parabolic trajectory results (this case is covered in the Software Tutorial Sheet, *Projectiles 1*). No allowance is made for aerodynamic drag and so the more realistic trajectory that is associated with projectiles ‘in the real world’ is not seen. The analysis of the more sophisticated case (included in the tutorial sheet, *Projectiles in a Resisting Medium. The Mathematics*) in which viscosity is included is an excellent exercise in the use of (a) vector components, (b) Newton’s 2nd Law and (c) first-order differential equations and their solution. This software tutorial, relating to the case in which the resistance to motion is *directly proportional* to the projectile’s velocity, should help deepen your understanding of how the trajectory of the projectile varies as parameters are changed; namely, mass, ‘viscosity’, launch height, velocity and angle of elevation. It is also possible to investigate the trajectory over an inclined plane.

Loading the Software

Load the “Projectiles” applet from the web site from which this document was taken. When loaded, you will see something similar to the applet window shown here.



Running the applet

The software default values are:

- *Height* = 0 metres, (height from which projectile is launched)
- *Speed* = 30 m/s, (initial speed and ...)
- *Elevation* = 65° , (... angle to the horizontal at which body is projected)
- *Mass* = 50 kg, (mass of the projectile)
- *Viscosity* = 0, (of the medium through which projectile passes – this is not in absolute units but is a factor that varies from zero to ten in order to obtain a *feel* for what happens at different viscosities)
- *Incline* = 0° , (this allows you to project the body across an inclined plane)

Print and use this sheet with the 'Projectiles' applet. You'll then have a permanent record of your work.

The six slider bars on the right of the applet window allow the user to change each of the above parameters. Clicking the arrow boxes at either end of the slider bar effect *small changes* in the parameter values. Clicking in the main part of the box of the slider bar makes *larger changes*. *Clicking, holding and moving the slider in the slider bar can make **any** sort of change.*

Changing parameter values by moving the sliders is the only means of user-input

The software gives you the option of showing single plots or multiple plots (under 'Plots' in the pull down menu structure at the top of the applet window). It also allows you to 'drag' the plot around (to see areas not covered by the original window on the plot) by holding down the mouse button whilst at the same time moving the mouse over the plot. The plot can be recentred at any time using the 'Centre on Origin' option, under 'Display'. You can reset the software at any time to the default values (those used when the program loads – given above) using 'Reset' from the 'Plots' menu.

The Tutorial

Changing viscosity Use the default values and sketch the trajectory on the axes below. Label your graph *Viscosity = 0*. Choose “Multiple Plots” from the “Plots” pull-down menu at the top of the applet window. Now use the applet to superimpose, by clicking ten times inside the viscosity slider bar to the right of the slider itself, the trajectories for which *Viscosity = 1 to 10*. Sketch the curves on the axes below, labelling the curves for which the viscosity values were 5 and 10.



Bearing in mind that changing viscosity represents projection through different fluids (not just air), use the eleven curves above to help comment on how viscosity affects trajectory.

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In particular, describe what happens to the maximum height and the horizontal range.

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In what way(s) does the curve for *Viscosity = 0* differ from the other cases? (See *Projectiles, Tut Sheet 1*)

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Press "Reset" and "Single Plot" from the "Plots" menu and change *Height* to a value of 40 by clicking in the slider bar to the right of the slider until a value of 40 is reached. Change the mass to 1 by clicking, holding and sliding the *Mass* slider completely to the left end of the slider bar (1 is the minimum value you can use). What happened to the trajectory as you changed the mass? (Notice that for these cases, the viscosity was equal to zero.)

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Now watch the graphics as you increase the value of the viscosity by clicking and holding down on the arrow at the right-hand end of the viscosity slider.

What can you see happening in the latter part of the trajectory as the value of viscosity increases to about 1.0 and above? (You may want to move the viscosity slider using the arrows at either end of the slider bar in case you didn't quite see this effect first time)

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So what is happening to the horizontal component of velocity in the cases for which viscosity is not equal to zero? (You can see the effect on the horizontal and vertical components of velocity if you 'turn on' the velocity vectors by selecting "Show Velocity" from the "Display" menu. You'll need to choose a suitable time interval from the supplied list. The vectors can be turned off subsequently from the same menu.)

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What happens to the horizontal component of velocity in the case when *Viscosity* = 0?

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Press "Reset" and "Single Plot" from the "Plots" menu and choose "Show Max Height" from the "Display" menu. This is the case of zero viscosity. Write here

initial speed = m/s

impact speed = m/s

horiz distance to max ht = m

Range = m

time to max. ht = seconds

time of flight = seconds

Within the limits of rounding errors, what do you notice about these two times?

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In other words, it takes as long for the projectile to come down as it does to go up.

In order to see the effect that viscosity has on the trajectory times and distances, change *Speed* to 60, *Mass* to 10 and *Viscosity* to 6 (and keep *Elevation* at 65 and *Height* and *Incline* at zero). Write here:

initial speed = m/s

impact speed = m/s

horiz distance to max ht = m

Range = m

time to max. ht = seconds

time of flight = seconds

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The time from when the projectile attains maximum height to the point where it hits the ground is, therefore, seconds

From the diagram it can be seen that the distance travelled from the point of projection to the maximum height is greater than the distance travelled from the maximum height back to the ground - and yet the times don't seem to agree with this. Why not?

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Changing Mass Use the default values for initial height, speed, etc by pressing "Reset", "Single Plot", "Centre on Origin" and "Show Max Height" (if necessary) from the relevant pull-down menus. Now move the viscosity slider totally to the right, resulting in a value of *viscosity* = 10 ('very' viscous case) and move *Mass* to a 'light' mass of *Mass* = 10. Note the large differences in the time to maximum height and the time of flight (corroborating what you found above). Superimpose, by using the *Mass* slider bar and "Multiple Plots", the four trajectories for which *Mass* = 20, 30, 40 and 50. Sketch the five curves on the axes below, labelling each with the appropriate *Mass* value.



Describe the effect of changing the value of *Mass*.

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Bearing in mind that the viscosity and the initial velocity were not changed - only the mass varies in all five cases - why do you think increasing the mass makes the object travel further?

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Galileo's famous experiment in which objects of various mass were dropped from the top of the Leaning Tower of Pisa can be simulated here. Choose "Reset", "Single Plot", "Centre on Origin" and "Hide Max Height" (if necessary). Put *Speed* = 0 and the angle of elevation to -90 (i.e. the object is *dropped* vertically downwards - although with *Speed* = 0 this is not absolutely necessary. Think about it!). Choose *Viscosity* = 10 and initial height 30. Superimpose cases for which only *Mass* changes. In particular, try values of *Mass* = 10, 20, 30, 40 and 50. Make a note for each case in the spaces below, the times that the object takes to fall the 30 metres.

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10..... 20..... 30..... 40..... 50.....

Galileo's conclusion was 'different weights fall the same distance in equal times', why do you think your conclusion here differs from Galileo's.

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Just to reinstate your faith in maths, repeat the exercise, but with *viscosity* = 0 (think about the physical situation this represents) noting the times below.

10..... 20..... 30..... 40..... 50.....

Why, on the basis of these results, did Galileo come to the wrong conclusion?

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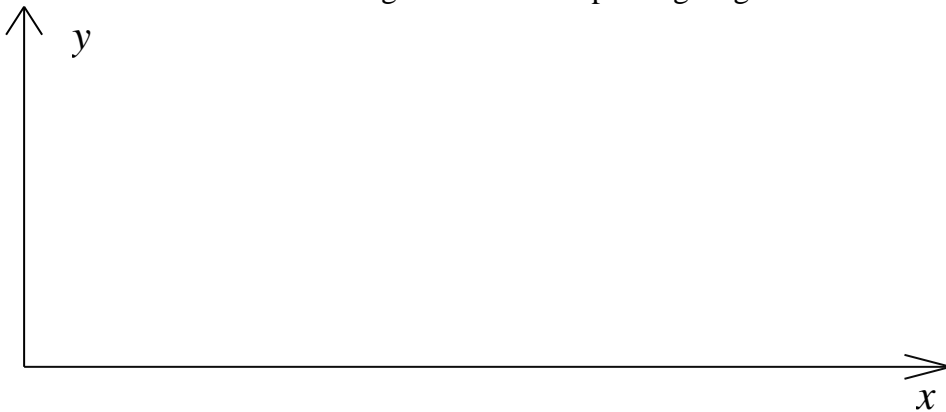
What does this tell you about the magnitude of the *viscosity* factor used in this software for objects dropped through air?

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So, under what condition is Galileo's conclusion absolutely valid?

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Changing initial height Use the values *Mass* = 10, *Viscosity* = 4.0, *Height* = 0, *Speed* = 30 and *Elevation* = 40 and sketch the resulting trajectory on the axes below. Use "Multiple Plots" and change *Height* to 10, 20 and 30. Sketch the trajectories below and indicate the values of the initial height and the corresponding range on the horizontal plane.



Describe the effect on the range of projecting the body from higher positions.

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Is this always going to be the case? For the viscous case, what happens to the range as the mass increases without limit? (To help you answer this, just extend the previous plots on your diagram with the cases $Height = 40, 50, \dots 100$)

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Changing initial velocity Use the slider bars to set up $Mass = 10, Viscosity = 5, Height = 0, Speed = 40$ and $Elevation = 60$, "Clear Multiple Plots" and superimpose the curves for which $Speed = 50, 60$ and 70 . Sketch the trajectories below and indicate the values of the initial velocities and the corresponding range on the horizontal plane.



Describe the effect on the range of projecting the body at different speeds.

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Is this always going to be the case? If so, why? If not, why not?

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(To help you answer the previous question, try varying $Speed$ for each of different $Mass$ and $Viscosity$ values and note how the range varies for these cases.)

Changing angle of projection In 'classical' projectile problems (where aerodynamic drag is not taken into account), the *maximum* range on a horizontal plane can be shown to occur when the angle of projection is 45 degrees. Do you think that this is always going to be the case for projectiles in a resisting medium? Yes / No

To test your answer, try the following. Ensure "Multiple Plots" is turned on. Use the slider bars to set up $Mass = 4, Viscosity = 1.5, Height = 0, Speed = 50$ and $Elevation = 41$ and use "Clear Multiple Plots". Now superimpose the curves for which $Elevation$ ranges from 41 to 46. Did you spot a max. range at 45°? Write down the ranges here.

Max range at 45°? Yes/No
 41..... 42..... 43..... 44..... 45..... 46.....

Use the arrows at either end of the Elevation slider bar to determine the angle in this case that provides the maximum horizontal range.

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Max Range of metres occurs when elevation is degrees.

Will this angle of elevation always give maximum range? Should you really extrapolate from the results of one test? Repeat the previous experiment but with $Mass = 100$ and $Viscosity = 10$. This time,

Max Range of metres occurs when elevation is degrees.

So, in the non-viscous case, the maximum range always occurs when the angle of elevation is 45° for all initial velocities, masses, etc. However, in the viscous case, maximum ranges can occur for angles of elevation other than 45° .