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The Straight Line, *y = mx + c*.
Tutorial Sheet
Learning Aims

- To develop, through experimentation, an understanding of the equation of the straight line and a feel for the effect of changing its parameters.
- To enable "what if ... ?" questions about the straight line to be answered.

Introduction

The straight line has important applications in the mathematical analysis of topics in, amongst others, science, technology and business studies. In particular, it is useful in areas in which one variable changes its value in ***direct proportion*** to another. For example, the extension of a spring fixed at one end varies directly with the weight of a mass suspended form its free end (within limits, of course) - the bigger the weight, the greater the extension. In an electrical circuit, Ohm's Law says that, for fixed resistance, voltage is directly proportional to the current. In these cases, plots of extension against weight and voltage against current result in straight-line graphs.

In Business Studies, it is often necessary to determine whether there exists a linear relationship (measured by the 'Correlation Coefficient') between two sets of variables. If this is the case, you have to determine the equation of the 'straight line of best fit' through the given data. For example, you may be trying to determine whether there exists a direct relationship between, say, sales of umbrellas and monthly rainfall. Your data would not all lie exactly on a straight line, but may be clustered so that there appears to be a general linear trend - and you would be trying to determine the equation of this straight line of best fit through your data.

This applet allows investigation of the general form of the straight line: *y = mx + c*

Loading the Software
Select "Maths Functions", followed by "The Straight Line" and then the "Run Applet" link from the list on the right hand side of the web page from which this document was downloaded.

Running the applet
When loaded, you are presented with the applet window as shown here. The software uses the values *m* = 1 and *c* = 0 as default values. With these default values, the relevant graph (in this case *y = x*) is displayed in the main graphics area as shown above.
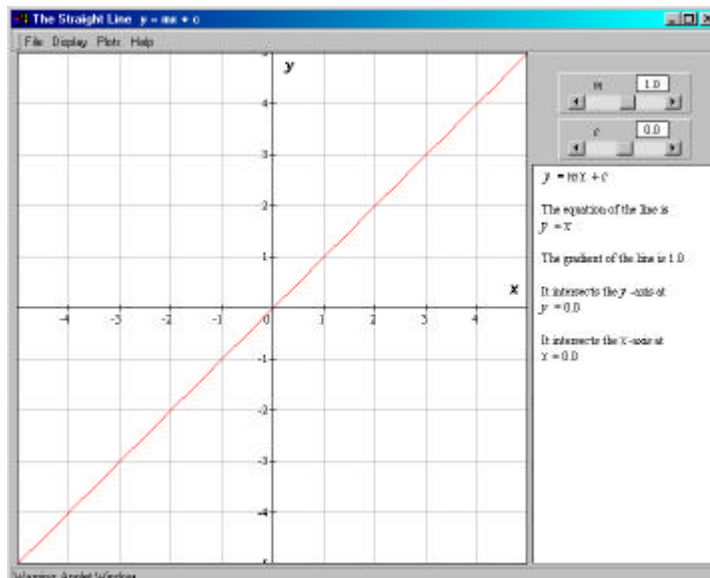
The two slider bars on the right of the applet window allow the user to change either of the parameters of the straight line (here *m* or *c*). Clicking the arrow boxes at either

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end of the slider bar effect *small changes* in the parameter values. Clicking in the main part of the box of the slider bar makes *larger changes. Clicking, holding and moving the slider in the slider bar can make any sort of change*.

Changing the values by moving the sliders is the only means of user-input.

The software gives you the option of showing single plots or multiple plots (under 'Plots' in the pull down menu structure at the top of the applet window). It also allows you to 'drag' the plot around (to see areas not covered by the original window on the plot) by holding down the mouse button whilst at the same time moving the mouse over the plot. The plot can be recentred at any time using the 'Centre on Origin' option, also under 'Plots'.

The Tutorial

Changing *m*

- With the default values $m = 1$ and $c = 0$, sketch the graph that you see on-screen on the axes below and write alongside this line its equation, in this case $y = x$. (This has already been done for you.)
- Choose "Multiple Plots" from the "Plots" pull-down menu and change the value of *m* to $m = 2$ by clicking once *inside* the *m* slider bar to the right of the slider itself.
- Repeat so that your plot shows also the graphs for $m = 3$, $m = 4$, $m = 5$ and $m = 6$.
- Superimpose all the lines on the axes below, labelling each line with its equation.

Where did each of these straight lines cross the *y*-axis?

....
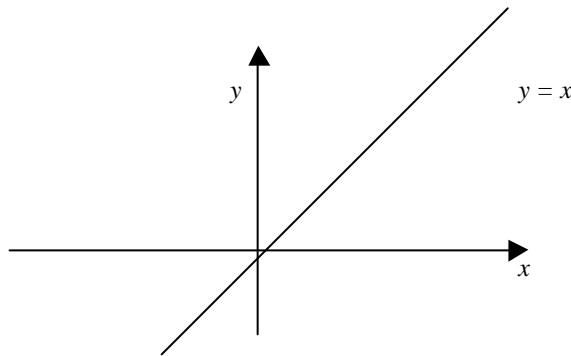
What was the effect on the line of increasing *m* from 1 to 6?

....

- Reset the graphic display to the default setting by choosing "Reset" from the "Plots" pull-down menu.
- With the default values $m = 1$ and $c = 0$, sketch again the graph that you see on-screen on the following axes and label it $y = x$. (Don't forget to label the axes, too)
- If not already chosen, choose "Multiple Plots" from the "Plots" pull-down men and change the value of *m* to $m = 0.9$ by clicking once *on the arrow* at the left-hand end of the *m* slider bar.

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- Repeat this so that your plot shows also the graphs for $m = 0.8$, $m = 0.7$, $m = 0.6$ and $m = 0.5$.
- Superimpose all the lines on the axes below, labelling each line with its equation.

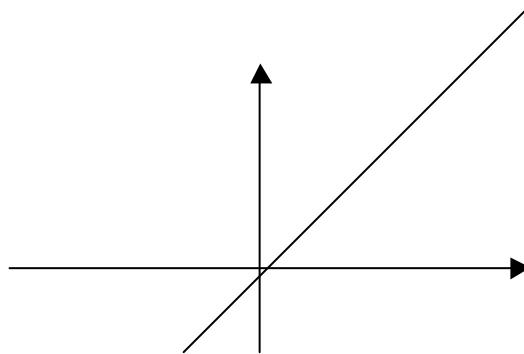
Where did each of these straight lines cross the *y*-axis?

....

What was the effect on the line of decreasing *m* from 1 to 0.5?

....

So on all plots so far, what was changing as you changed *m*?

....

... and what didn't change?

....
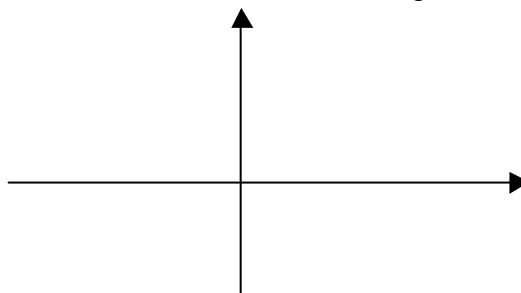
In the light of what you have found so far, what do you guess would be the effect of making the value of *m* equal to zero?

....   Try it now.

- Click on the slider of the *m* slider bar, hold down the mouse button and move the mouse to the left until the value of *m* is -1
- You may need to clear all other plots on-screen before proceeding. This is done using "Clear Multiple Plots" from the "Plots" pull-down menu.
- Sketch again the graph that you see on-screen on the following axes and label it (and label the axes too).
- If not already chosen, choose "Multiple Plots" from the "Plots" pull-down menu and change the value of *m* to $m = -2$ by clicking once *inside* the *m* slider bar to the left of the slider itself.
- Repeat this for $m = -3$, $m = -4$, $m = -5$ and m $= -6$.
- Superimpose all the lines on the axes below, labelling each line with its equation.

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Where did each of these straight lines cross the *y*-axis?

....

What was the effect on the line of decreasing *m* from -1 to -6?

....

What is the overall effect of making the value of *m* negative?

....

Superimpose on the previous axes the graphs for which $m = -0.9$, $m = -0.8$, $m = -0.7$, $m = -0.6$ and $m = -0.5$ (all with $c = 0$). Use the applet if you can't guess.

Why do you think that the applet stops at $m = 10$ in the positive direction and $m = -10$ in the negative direction?

....

What do you think would be the 'value' of *m* for a vertical line?

....
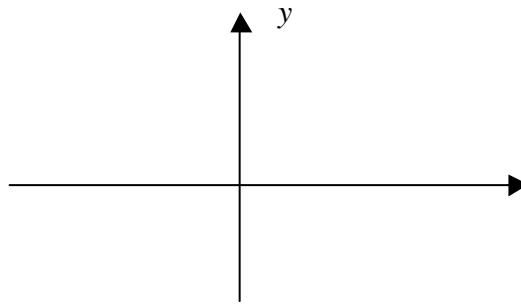
Changing *c* Now use the slider bars to set up $m = 1$ and $c = 2.5$ and clear the screen of any multiple plots. Superimpose on-screen the lines for which $c = 5.0$, $c = 7.5$, ..., as far as you need in order to understand the effect of increasing *c* and to sketch the obtained family of lines on the following diagram. Indicate on your diagram the value(s) of the *y*-intercept(s) and label each line.
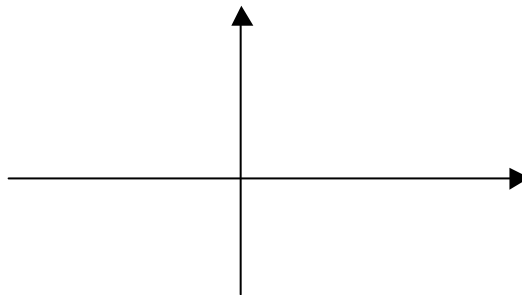
Describe the effect of increasing the value of *c*. What happened to the slope during this time? Why?

....
....………......
....
....………...................
....

Now consider *c* with negative values. Try $c = -2.5$, $c = -5$, $c = -7.5$, Sketch and label the curves along with their *y*-intercepts on the next page.

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Did you notice that you haven't tried $c = 0$? Before using the applet, write below what you think is special about any lines with this value of c.

....

Describe in the space below the line obtained when both $m = 0$ and $c = 0$? Now use the applet plot it and see.

....

Exercises

1. *Without* using the software, sketch[⊕] the following straight lines:

$y = x$	$y = x + 1$
$y = 2x$	$y = 2x - 3$
$y = -3x + 4$	$y = -3x - 4$
$y = 0.5x - 5$	$y = -0.5x + 5$

   Verify your sketches using the software.

2.   (i) Show that the equations
          $2x - y = 3$ and
          $x + 2y = 4$
       are both linear equations (i.e. can be expressed in the form $y = mx + c$)

      (ii) Use the software to plot both of these lines and try to estimate the co-ordinates of the point where the two lines cross (intersect).

      (iii) Solve the above two equations simultaneously.

      (iv) What is the relationship between the answers you found in parts (ii) and (iii)?

3.   Write down the equation of the line that
      i.   has slope 3, intercept 4
      ii. has slope -0.3, intercept -3.2
      iii. is horizontal crossing the *y*-axis at $y = 4$
      iv. is vertical passing through the point (5,2)
            [Note: you are not allowed to use the symbol '∞' in an equation]

[⊕] Here 'sketch' means roughly draw the axes with the straight lines superimposed. You should label the axes and each graph and indicate any important points (such as the *y*-intercept). 'Plot', on the other hand, means 'plot points accurately on a properly scaled and labelled graph' using data from a table of values. You should label the graphs and important points in either case.

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Some Final Comments / Theory

The equation $y = mx + c$ is the equation of a ***straight line.***

x and y *are* **variables,** since for a given straight line, their values vary.

x is called the *independent variable* (it is usual to choose values for x *arbitrarily*). However, once x has been chosen the value of y becomes fixed by the linear equation. The value of y obtained is *dependent* on the value of x used, so y is called the *dependent variable.*

When drawing graphs of equations in which y depends on x, the vertical axis is always labelled with the dependent variable (here y) and the horizontal axis is always labelled with the independent variable (here x).

m and c are **parameters.** Although it is also possible to choose values for m and c arbitrarily, for any particular straight line their values will not change. A new set of values for m and/or c will result in a *different* straight line. m represents the ***slope*** of the line and c represents the ***y-intercept*** (the y-coordinate of the point at which the straight line crosses the y-axis - when the y-axis is placed at $x = 0$).

On a straight-line graph, a line that *rises from left to right* is said to have a ***positive slope.*** A line that *falls from left to right* is said to have a ***negative slope.***

If y is a function of (i.e. depends on) x, it can be written $y = f(x)$ in general. For the straight line in particular, $f(x) = mx + c$. Here, $f(x)$ is said to be a ***linear function.***

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